D Bradfield*

Investment Basics XLVI. On estimating the beta coefficient

1. INTRODUCTION

The estimation of the beta coefficient has traditionally been achieved by running a Market Model regression. Running this regression can, however, lead to a variety of practical considerations which in turn could result in several different beta estimates. Some of these could be purely measurement related such as: How does one measure returns? What market proxy should be used? How long should the return intervals be? How many data points are needed? On the other hand, a further set of considerations involve the assumptions and the inferences: such as: Is thin trading a problem? Is the market segmented? Are betas likely to be stable?

For practitioners wanting to estimate betas on the JSE, this article gives guidance on the steps needed to be taken to ensure the resulting betas are accurate. For users of published betas, it gives readers an understanding of the care needed in the estimation of beta coefficients.

2. THE MARKET MODEL

The basic concept of beta arises because all stocks tend to move to some extent with movements in the overall market. Clearly some stocks tend to move more than others when the market moves; hence their sensitivity to movements of the overall market index is an important measure - widely known as the beta coefficient. The Market Model has traditionally been used to estimate the beta coefficient:

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it} \qquad \dots (1)$$

where

R_{it} is the return on asset i at time t,

R_{mt} is the return on the market (or benchmark) at time t,

α_i and β_i are the intercept and slope (beta) coefficients to be estimated for asset i.

The market model is commonly estimated using ordinary least squares regression (OLS). In this instance the OLS estimate of beta is simply:

$$\beta_i = \frac{\mathrm{cov}(R_{it};R_{mt})}{\mathrm{var}(R_{mt})} \qquad \dots (2)$$

It should be noted that the market model is not based on any assumptions about investment behaviour but simply posits a linear relationship between stock returns and the market return. Figure 1 and Figure 2 show the scatter diagrams for the returns on two stocks Anglo American and SAB Miller, regressed on the returns of the All Share Index (ALSI) respectively. It is evident that the estimated (OLS) beta for Anglo American is 1.34 and for SAB Miller is 0.96, indicating their differing sensitivities to market movements.

It should be noted that whilst this article focuses on the estimation of betas on individual stocks, the concept could be applied to any asset, including investment portfolios.

3. RETURN MEASURES

The first consideration is the construction of the return series for both the stock and the market index used.

The user has a choice, either discrete or continuously compounded returns can be used, as long the consistency between the asset returns and the market index proxy is maintained. It is generally accepted that returns are continuously generated through calendar time. However because trading occurs at discrete intervals, observers view returns as if they are generated at discrete intervals (see for example Brailsford, Faff and Oliver (1997)).

For both the discrete and continuously compounded returns it is important to note that the returns should be adjusted for capitalization changes and dividends.

4. THE MARKET INDEX

In theory market capitalization weighted indices are preferred to equally weighted indices because they are superior proxies to the true market portfolio. Hence in South Africa, the All Share Index (ALSI) should be used. It should be noted that some practitioners argue that there is a perceived segmentation between the Resources and Financial and Industrial sectors on the JSE and consequently prefer to use the Financial and Industrial Index as an overall market proxy for stocks belonging to this category. It should however be noted that the received theory calls for an index that is as comprehensive as possible in covering the market.

*Cadiz Holdings Limited, 1 Oakdale Road Newlands, Cape Town, Republic of South Africa.
Email: david.bradfield@cadiz.co.za

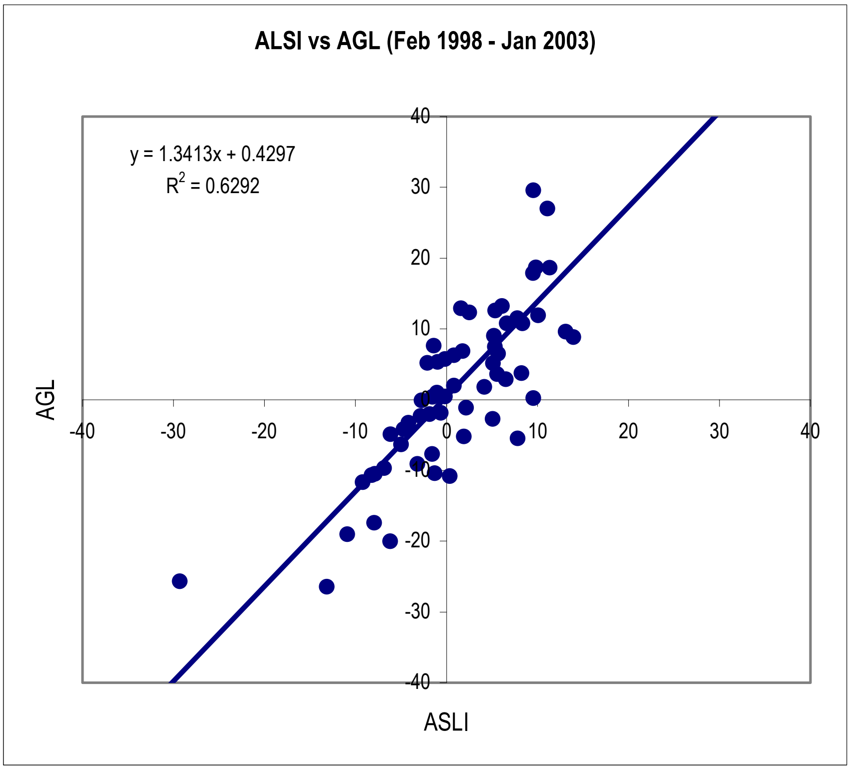

Figure 1: Scatter diagram for Anglo American

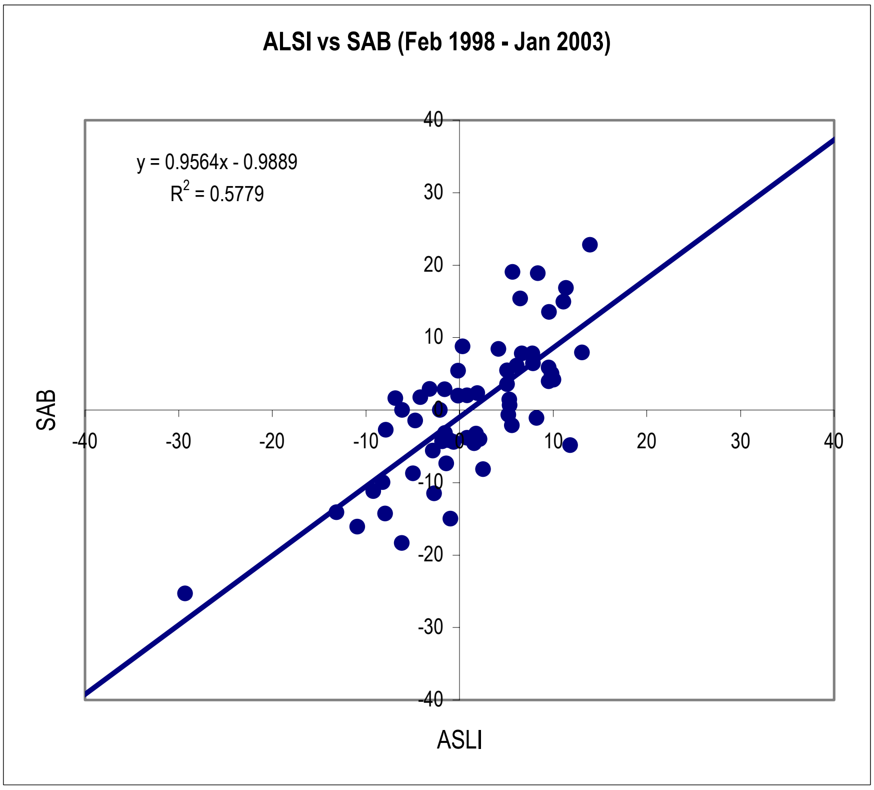

Figure 2: Scatter diagram for SAB Miller

5. LENGTH OF THE ESTIMATION PERIOD

Estimates based on many years of historical data may be of little relevance because the nature of the business risks undertaken by companies may have changed significantly over a long period such as 10 years. The choice of a five-year estimation period is based on the findings that betas tend to be reasonably stable over five yearly periods (see for example Gonedes (1973) and Kim (1993)). The selection of a five-year period represents a satisfactory trade-off between a large enough sample size to enable reasonably efficient estimation and a short enough period over which the underlying beta could be assumed to be stable.

6. THE RETURN INTERVAL

Pogue and Solnik (1974) were the first to measure the impact on the estimates of beta obtained by using different return interval lengths. Much research has been directed at establishing the impact that different interval lengths have on estimates of beta. Subsequently Blume (1975), Eubank and Zumwalt (1979) and more recently Corhay (1992) have assessed the effect that various interval lengths have on the predictive power of beta estimates.

As a consequence, researchers use monthly intervals (over a five year period) to compute the returns needed for the estimation process, resulting in 60 data points of monthly returns .

7. CORRECTING FOR THE REGRESSION BIAS USING A BAYESIAN ADJUSTMENT

Blume (1971, 1975) was the first to document that individual stock betas had a regression tendency towards the grand mean of all stocks on the exchange. This regression bias can be described as follows: an estimated beta coefficient which is far higher than the average beta is more likely to be an overestimate of the true beta than an underestimate. Similarly, a very low estimated beta is more likely to be an underestimate. Thus the estimates of beta obtained from the regression analysis may be suboptimal for forecasting purposes. To correct for the regression bias, a "Bayesian" adjustment of the form suggested by Vasicek (1973) can be implemented. Hence betas can be corrected as follows:

$$\hat{\beta} = w\beta_{OLS} + (1 - w)\bar{\beta}_{OLS} \qquad \dots (3)$$

$$w = \frac{\sigma_\beta^2}{(\sigma_\beta^2 + \sigma_{OLS}^2)} \qquad \dots (4)$$

where

β_{OLS} is the OLS estimate of beta

$\bar{\beta}_{OLS}$ is the average OLS beta of all stocks in the market

σ_{OLS} is the standard error of the OLS estimate of beta

σ_β is the cross-sectional standard deviation of all the estimates of beta in the market

It can readily seen from the above formulation (4) that the weight , w , assigned to the OLS beta will be large if the cross-sectional standard deviation (σ_β), is large. In other words, if the spread of the betas across the stocks is so large as to make all values of beta equally likely, then the OLS beta estimator is optimal. Conversely, if the standard error of the OLS beta estimate, σ_{OLS} , is large relative to σ_β , then w will be small and the Bayesian estimate of beta will be "shrunk" towards the overall average beta of the stocks on the market. Therefore an estimate of beta which falls outside the usual spread of beta (and which has a large standard error) is likely to be an overestimate. Hence the above expression corrects for the commonly observed phenomenon that very high beta estimates that are unreliable (large standard errors) tend to be overestimates, and very low betas that are unreliable tend to be underestimates.

A variety of beta services use this Bayesian approach to adjust for this regression tendency. However many assume a constant weighting scheme to shrink the betas across all the stocks, and consequently shrink all estimates independent of how unreliable they are.

8. ADJUSTING FOR THIN-TRADING

The bias in beta estimates caused by thin-trading on the JSE has been documented by Bradfield (1990). If a stock is thinly traded then it is likely that the month-end price may not arise from a trade on that day but may instead be recorded as the price last traded during the month. Consequently the recorded price on the market index at month-end may not be matched to a trade for the stock on the day – hence a mismatch occurs. This mismatching phenomenon clearly has an impact on the covariance estimate between the stock and the market proxy, leading to a downward bias in this covariance estimate. The thin-trading bias manifests itself in the OLS beta estimate because the OLS estimate of beta has this covariance term in the numerator (see equation (2)). Hence the downward bias in the covariance estimate caused by thin-trading translates into a downward bias in the estimate of beta.

Several researchers have devised techniques for obtaining unbiased estimates for beta in infrequently traded environments. Two distinctly different approaches have emerged; the "trade-to-trade" estimator (discussed for instance by Marsh (1979), Dimson (1979) and Bowie and Bradfield (1993) and the Cohen estimators (which encompass all methods which use non-synchronous coefficients, e.g. Scholes and Williams (1977), Dimson (1979), Cohen, Hawawini, Maier, Schwartz and Whitcomb (1983)).

The Cohen type of estimators are based on aggregating lagged and leading regression coefficients whilst in the trade-to-trade approach the returns are matched and measured during the last consecutive trading days in each month. Bowie and Bradfield (1993) assessed the superiority of these two types of estimators on the JSE and conclusively found that the trade-to-trade was superior for application on the JSE. They found that the standard errors of the trade-to-trade technique were substantially smaller than those of the Cohen estimator, emphasizing that the trade-to-trade estimator is more efficient.

In the trade-to-trade method the returns on the stock and on the index are measured between the times of the last trades in successive months. Thus a statistical correction needs to be made for potential durations in return no longer being equal. Furthermore to improve the efficiency a correction is required for the heteroscedasticity in the residual component. This leads to the final trade-to-trade estimator proposed by Dimson and Marsh (1983):

$$\frac{R_{it}}{\sqrt{D_{it}}} = \alpha_i \sqrt{D_{it}} + \beta_i \frac{R_{mt}}{\sqrt{D_{it}}} + e_{it} \qquad \ldots (5)$$

where

D_{it} is the proportion of a month between successive traded months for stock i and month t.

R_{it} are the returns computed over the last traded day in each month,

R_{mt} are the market returns matched to the same consecutive traded days as stock i.

Note that the above estimate of beta can replace the OLS beta in the Bayesian correction if the two correction procedures are used concurrently.

9. OTHER ISSUES RELATING TO THE USE OF BETAS

9.1 The stability of betas

Detection of beta stability is clouded by the fact that only the estimates of beta are observable. Changing estimates do not necessarily imply stable underlying betas. Research by Bowie and Bradfield (1997) find that results of tests on the stability of betas are difficult to interpret on their own but conclude that the JSE betas are as stable as betas of stocks on the UK market.

9.2 Robust estimation of beta

Bowie and Bradfield (1998) assessed the efficiencies of a wide range of robust estimators on the JSE. The study indicates that the robustness required for estimating betas involved down-weighting not only the outlying residual values, but also the outlying market returns. They find that the bounded influence estimator Lp-norm to be the superior robust estimator on the JSE.

9.3 Dynamic estimation of beta

Filtering and smoothing techniques can serve as useful aids in the interpretation of time series of return observations. Techniques are available that categorize the movements in the levels of stock returns as either permanent shifts in the riskiness of the firm, or as one-off events attributable to specific circumstances. In South Africa Bowie (1994) examined several techniques that yielded promising results in terms of the ability of estimates to respond to sudden changes in the riskiness of a stock. Bowie (1994) points out that a problem arises when using the well-known Kalman-filtering approach to dynamically estimate betas because of the potential non-Normality of stock returns on the JSE. Instead he recommends a filtering approach proposed by Masreliez (1975).

10. USING A "BETA BOOK"

There are many services that supply betas and associated estimates arising from the Market Model.

We have included an extract from the Financial Risk Service (September 2002) published by the University of Cape Town. This service uses both the trade-to-trade correction procedure given in equation (5) as well as the Bayesian correction procedure (equation (3)).

The table represents the output from a typical beta service. The columns, which give the statistics emerging from market model regressions, are interpreted as follows:

Alpha: This is the average return (per month) on the share when the market on average does not move.

Std err(β): The standard error of beta is a statistical measure of the reliability of the estimate of beta. The lower this figure is the more reliable the estimate of beta. Statisticians set up a confidence intervals for the estimate of beta by adding and subtracting 2 x Std err(β) from the beta estimate. There is a 95% chance that the true beta lies in this interval.

Table 1: Extract from Financial Risk Service (September 2002)

Security	Code	No. of months	Alpha	Beta	Std err(β)	Total risk	Unique Risk	R^2	Days not traded
AFRIKANDER LEASE LTD	AFL	60	0,04	0,76	0,43	27,0	26,2	5,8	158
ANGLOGOLD LTD	ANG	60	0,02	0,76	0,20	13,1	11,7	20,8	0
ARMGOLD	AOD	5	0,12	1,14	0,76	13,7	8,0	65,7	1
AVGOLD LTD	AVG	60	0,02	0,53	0,29	17,3	16,8	5,8	4
DURBAN ROODEPT DEEP	DUR	60	0,05	0,81	0,43	26,0	25,1	6,7	0
EERSTELING GOLD MIN CO	ESL	60	0,05	0,84	0,59	35,9	35,1	4,3	953
FALCON INVEST SOC ANON	FLC	39	-0,03	0,50	0,46	20,7	20,4	3,1	1219
GOLD FIELDS LTD	GFI	60	0,02	0,65	0,21	13,2	12,1	15,2	0
HARMONY G M CO LTD	HAR	60	0,04	0,85	0,28	17,6	16,2	15,0	1
PETRA MINING LTD	PET	60	0,04	-0,11	0,37	22,6	22,5	0,5	984
PRESIDENT STEYN GOLD MIN	PGD	45	0,00	0,61	0,66	27,4	27,1	2,2	205
RANDGOLD & EXP CO LTD	RNG	60	0,02	1,21	0,33	21,6	19,2	21,4	80
SIMMER & JACK MINES LTD	SIM	60	0,05	0,45	0,70	42,3	42,2	0,6	837
STILFONTEIN G M CO LTD	STI	60	0,26	-0,55	1,73	115,2	110,4	8,2	964
SUB NIGEL GOLD MINING CO	SBN	60	0,06	0,54	0,79	45,4	45,3	0,8	533
VILLAGE MAIN REEF G M CO	VIL	60	0,00	0,12	0,19	11,4	11,4	0,5	885
WESTERN AREAS LTD	WAR	60	0,01	0,90	0,27	17,6	16,0	17,1	9

Total Risk: This is the standard deviation of returns measuring the share's total risk expressed in % per month.

Unique Risk (or Non-Systematic Risk): reflects the fluctuations in the security's returns that are linked to events unique to the company (e.g. bad management, worker strikes etc.).

R^2: This can be interpreted as the proportion of the share's total risk accounted for by its market risk. Note that a high beta will not necessarily produce a high R^2. In statistical terms, R^2 is the coefficient of determination of the regression.

Days not Traded: The number of days over the period of analysis during which the security did not trade. This provides an indication of the extent to which the security is thinly traded. Over the last 60 months, the JSE traded for 1248 days, about 21 days per month. If a particular security, for example, has been listed for five years and has "691 days not traded" recorded, then it has only been traded about 45% of the time (100 x [1 - 691/1248]). In some instances, for extremely thinly traded securities, more than five years of data is needed. In these cases it is possible that more than 1248 "days not traded" could be recorded.

11. SUMMARY

The primary aim of this article has been to provide guidance to the practitioner wanting to estimate betas. Research in South Africa has indicated the significant bias in beta estimates caused by thin trading as well as the regression tendency (for betas to revert to the mean). Thus of all the refinements briefly discussed here, the most important of these are the thin-trading correction (especially when it is known that stocks suffer from thin-trading) and the Bayesian correction.

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