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Financial Economics

Session 2: MPT and CAPM

Postgraduate Class

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Department of Economics

DEPARTMENT OF ECONOMICS

Introduction

- In this session, we will be following the theoretic discussion in chapter 11 of Ruppert (2011). *Statistics and Data Analysis for Financial Engineering*. Springer.

What we are going to do today…

- We will be looking at the standard financial theory models that developed and spread after the 1960s and continue to be the basis for much of what we know and have learned in Financial literature.
- This implies looking at Modern Portfolio Theory (MPT), and using it to deduce a **fair price** for assets in an efficient financial system.
- This implies the derivation of the CAPM model (graphically).
- We will then also critically discuss Asset Pricing Models, their applicability and how they are used by companies in South Africa.

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Valuing assets:
Net Present Value (NPV) models (constructing a normative pricing world)

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Asset Pricing…

- Consider this inherently unanswerable question:

How do we accurately determine the fair price of an asset?

- This might not seem obvious at first, but the straight answer is that we cannot. The price of an asset is essentially, like beauty, in the **eye of the asset-holder.**

Asset Pricing…

- Consider then this rational question that follows:

Why would we want to even try and measure something inherently immeasurable?

- Although this can get unnecessarily philosophical, let's suffice with the obvious: if we can reach a [even if imperfect] fair price for a specific type of asset, we can determine whether an asset is over- or undervalued – and let that be a quantitative guide for investment decisions!

Conceptually… what are we doing?

- **<u>Pricing theory</u>**, according to Cochrane (2001)'s *Asset Pricing* book, struggles with the same <u>positive</u> vs <u>normative</u> tensions faced in other areas of economics:
  - Should Pricing theories describe how the world ***<u>actually works</u>*** (positive), or how it ***<u>should</u>*** work (normative)
- In valuing assets, this tension is real.
- When observing an asset's price (e.g. Naspers shares) – should we approach valuation *positively* by understanding **why** prices are where they are… or normatively, arguing that the market's pricing might be wrong (i.e. current price is cheap or expensive, e.g.) – providing us with an investable opportunity.

Normative valuations

- Much of the finance industry (active) is premised on identifying stocks that are either undervalued (buying it) or overvaluing it (avoiding it if long-only, possibly shorting it if a hedge fund).
- Benjamin Graham (legendary stock investor) cautions on identifying an intrinsic value of stocks (despite arguing himself it should be attempted when investing)- stating it is “elusive” and cannot be determined definitively.
  - Consider that: Intrinsic value requires a set of (debatable) assumptions to hold, and can never be regarded as truth or generally applicable.
- Nonetheless, converging on an intrinsic (fundamental) value could be profitable if (…big if) markets agree and prices correct.

Normative valuations

- How could people then differ on pricing of assets?
- Core to asset valuations is the concept of risk (ability to deviate from expected returns).
  - Different investors have different risk profiles
  - This can (even in a highly efficient market) lead to different perceptions of risk
- Consider e.g. two investors:
  - A has an investment horizon of 1 year (wants to use cash for home deposit in 1 year)
  - B has investment horizon of 20 years (until retirement).
- Would A and B, assuming full rationality, value a share with high short-term risk similarly? Can an objective intrinsic value be derived that is the same for both of them?
  - **<u>NO! A would of course require a higher premium for investing in a risky asset…</u>**

Net Present Value Models

- Premise
  - All NPV models work on the basic principle that share prices accurately reflect future return streams, discounted to today's prices.
  - $PV = \frac{C_1}{1+r_1} + \frac{C_2}{1+r_2} + \frac{C_3}{1+r_3} + \cdots$
  - This is a fair assumption right?
  - You pay today for what you expect to earn (in cash, C) during the time that you hold the asset.

- Problems
  - Adding objective risk premium is not trivial (odds of not receiving C_t at time t, or of the stock becoming insolvent)
  - What to use for as the rate of interest?
  - Also → with stocks in particular, cash payouts are not mandated, nor easily forecastable for long periods…

Net Present Value Models

- Examples
- DDM (Dividend Discount Models)
  - Basically, the assumption is that a stock's current price is a function of all future dividend payouts discounted to today's value, plus whatever proceeds are accrued at the ultimate liquidation of firm (shareholders have a residual claim on assets – more on this later).
  - Assumption – current prices reflect future payouts accurately
  - Problem:
    - Have to forecast dividend payments in future, which is a function of future performance, governance structures and reinvestment needs.
    - …in addition to problems of defining interest rates for discounting

Net Present Value Models

- Examples
- DCF (Discounted Casfhlow Models)
  - Idea here is rather to consider cash-flow changes in future – emphasizing importance of cash-flow, regardless of whether it is ultimately paid out (as dividends) or not.
  - Problem: Only applies to companies with positive (and predictable) cash-flows.
  - This does not apply to all sectors of course…
  - Preferred by most over DDM due to its broader application and more intuitive appeal (as well as the fact that cash-flow is less open to manipulation by management)

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The Relative valuation of assets:

Using MPT and CAPM as the core theory underlying asset pricing strategies

Relative Valuations

- Pricing theory becomes significantly simpler and more manageable (arguably more applicable?) when we consider **relative valuations** as opposed to trying to find intrinsic fundamental values.
- In this section we look at valuing assets relative to their exposure to market risk.
- We will in a next session discuss other means of applying relative valuations using, e.g., EBITDA/EV, PE, EV/EBITDA, and other measures…

- For now, let's look at price and risk relatives using the MPT and CAPM model framework for identifying relative valuations.

SOURCES (for this session and the next)

- Perold (2004). *Capital Asset Pricing Model*
- Fama and French (2004). *Capital Asset Pricing Model: Theory and Evidence.*
- Morningstar MPT overview
- Bradfield, D. (2011). *Financial Risk Service On the JSE*. Cadiz Investment institution
- Jones (2008). *Chapter 4* (completely optional!)

MPT

- Modern Portfolio Theory has been attributed to Harry **Markowitz** who published several articles (the first: Portfolio Selection) in the 1950s which, together with subsequent contributions from other academics, set out to better explain how investment portfolios are constructed optimally.

- The main contribution of MPT to finance is that it showed how the age-old concept of **diversification*** can lower portfolio risk and by definition then impact individual asset prices.
  - *According to Herbison (2003), the proverb "Do not keep all your eggs in one basket" appeared in Italian writings dating back to the 1600s.

MPT

- Markowitz showed how diversification could be beneficial to investors by **combining imperfectly correlated assets** :
- Thus advising investors not only to put their eggs into different baskets, but more specifically baskets that have been shown to not be **perfectly correlated** (else a failure in all your baskets can happen at the same time!)
  - MPT also showed that certain broad economic (or market) specific factors were inherent in all assets within such an economy – splitting risk into a **diversifiable** and **non-diversifiable** risk component!

Insights that made MPT famous…

- First, we assume the following:
- Assets adhere to the **Random Walk principle**. This implies that asset prices in the market could at most follow a LR trend based on fundamental factors – but fluctuate in the SR **at random** from this trend…
  - This implies that the prediction of asset prices in the SR are essentially a fool's (or asset manager's) game.
  - This has been a hotly contested feature of MPT – which has seen a large body of literature develop to "debunk" it.
  - → Most point to the fact that if it was true, there'd be no traders in financial markets

MPT: Assumptions - EMH

- The Random Walk principle is based on the Efficient Market Hypothesis (EMH) of asset prices.
  - The hypothesis has three varying degrees of strength:
  - **Strong:** all possible information, both public and private and both future and historic, available today, is already incorporated into the price of an asset – implying that any price fluctuation is due to random events occurring in the future and which could not possibly have been predicted today - thus there is <u>no possibility of arbitrage</u> in buying and selling assets.
  - EMH therefore implies that there is **an even chance of over- or underperforming** the market when betting on an asset / market portfolio price.

The no-Arbitrage assumption

- Notice that EMH implies that no profit stands to be gained by merely exploiting inaccurate pricing (i.e. no free-financial-lunches are dished out in the Fin Market).

- Thus traders (or investors) in financial securities (who look to generate arbitrage returns) **play a fundamental role in feeding information into the market and so correcting asset prices,** yet the strong form of EMH suggests they would be lucky to profit off such strategies… and would’ve been impossible to know ex ante whether they’d beat the market or underperform

Can markets *actually* be regarded as efficient?

Valid questions to grapple with re market efficiency:

- If markets were truly efficient, why would trade exist?
- If markets were truly efficient, why do active managers exist?
- Is there a bubble in simple vanilla index strategies? – can the entire market be in ***vanilla*** form?
- Can the entire market be in ***indexation*** form?
- How will AI in finance affect market efficiency and the active vs indexation debate?

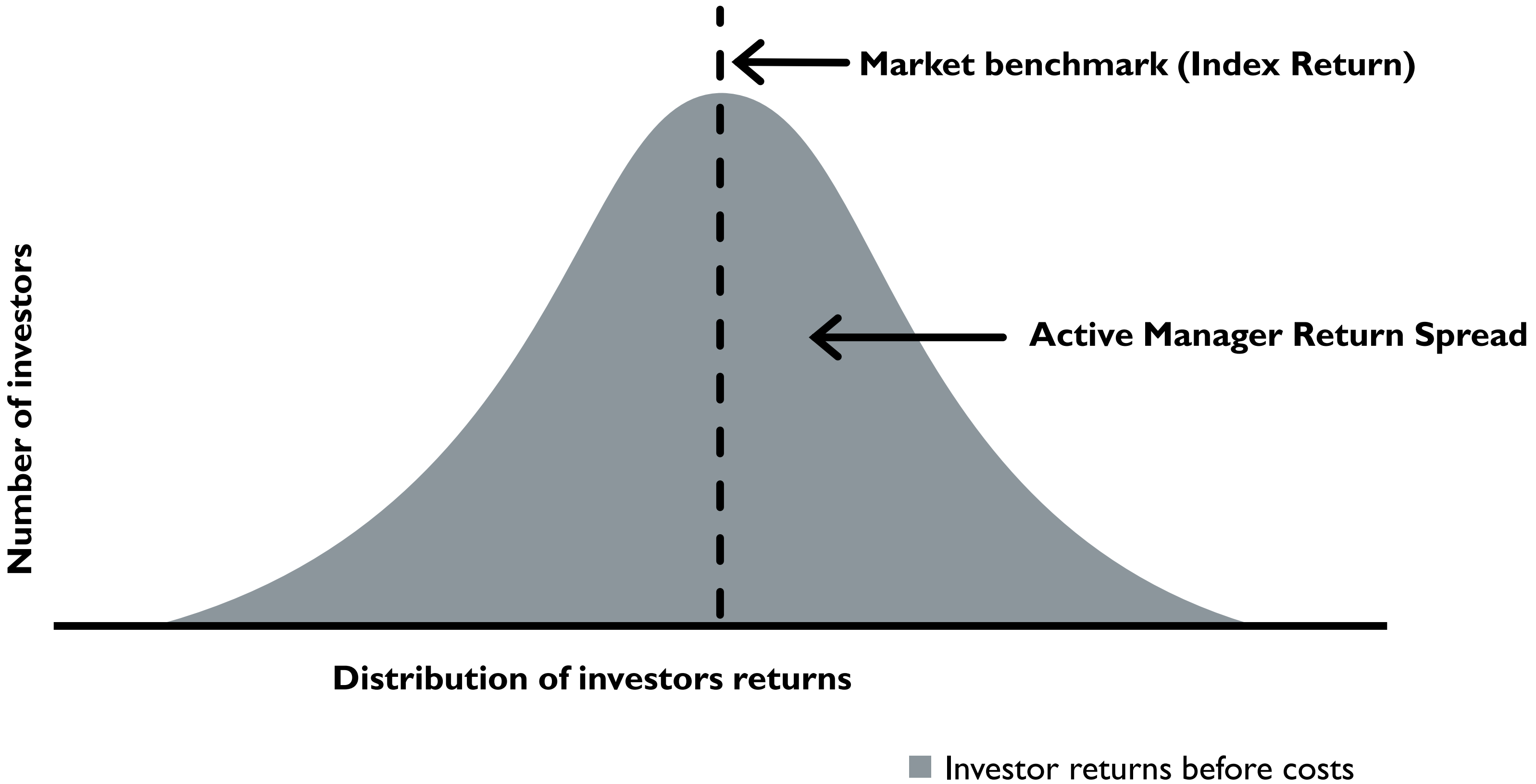
Quick digression on passive vs active…

β	*Refined* β	α
Vanilla Index Funds	**Non-Vanilla Index Funds**	**Actively Managed Funds**
tracks a market index	systematic strategies	conviction, research
•lowest expense ratios •transparent •low turnover	•low expense ratios •transparent •style consistency	•higher expense ratios •less transparent •varying styles
market performance	**able to beat the market**	**able to beat the market**
simple rules based	**refined rules based**	**active**

Classifying all indexes as passive makes no sense

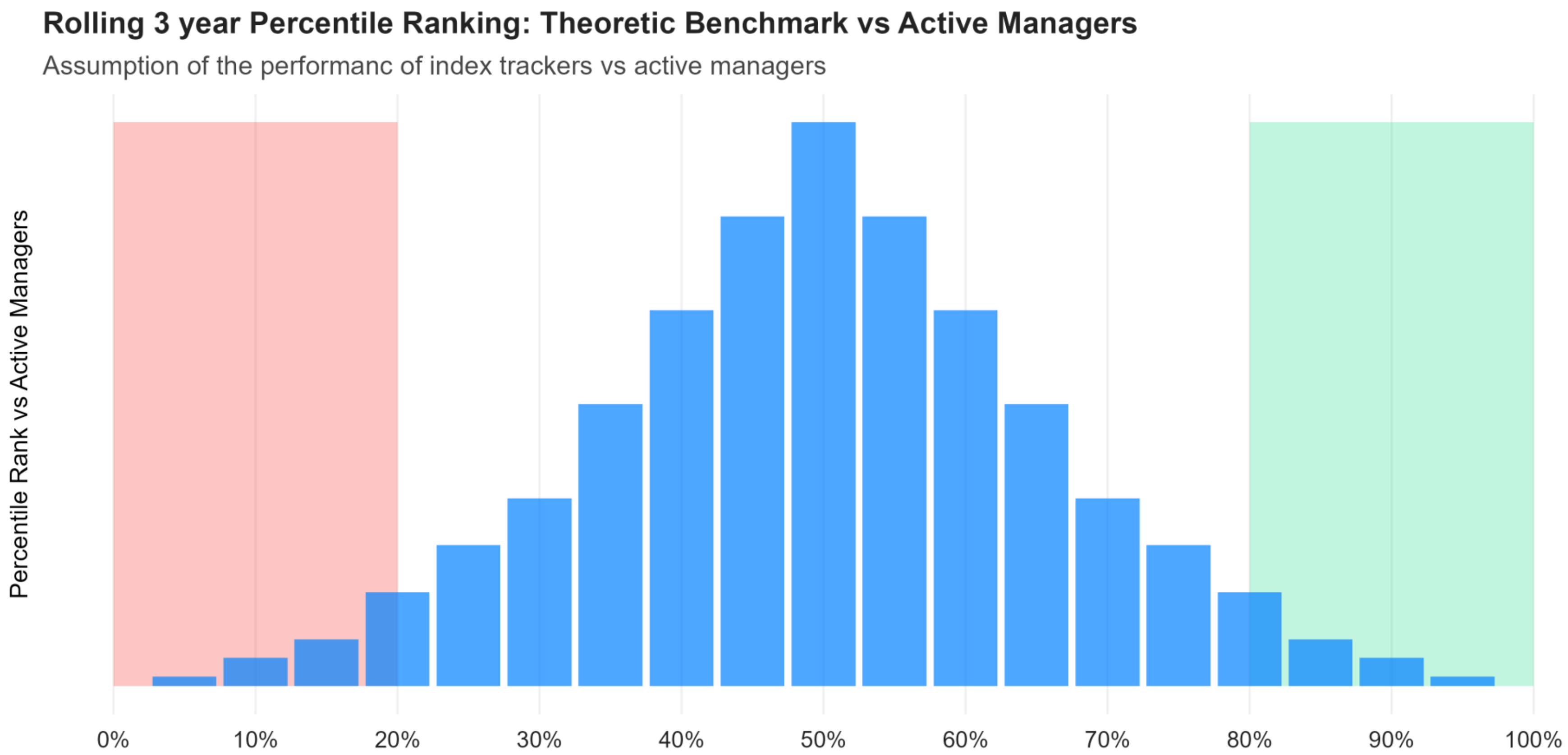
What we think indexation delivers relative to active peers…

We tend to think index trackers perform at the median…



Myth 2: Using vanilla index solutions in your portfolio is lazy

..but index trackers consistently outperform the median over three year periods…



Source: Satrix. Data: Morningstar, FTSE/JSE

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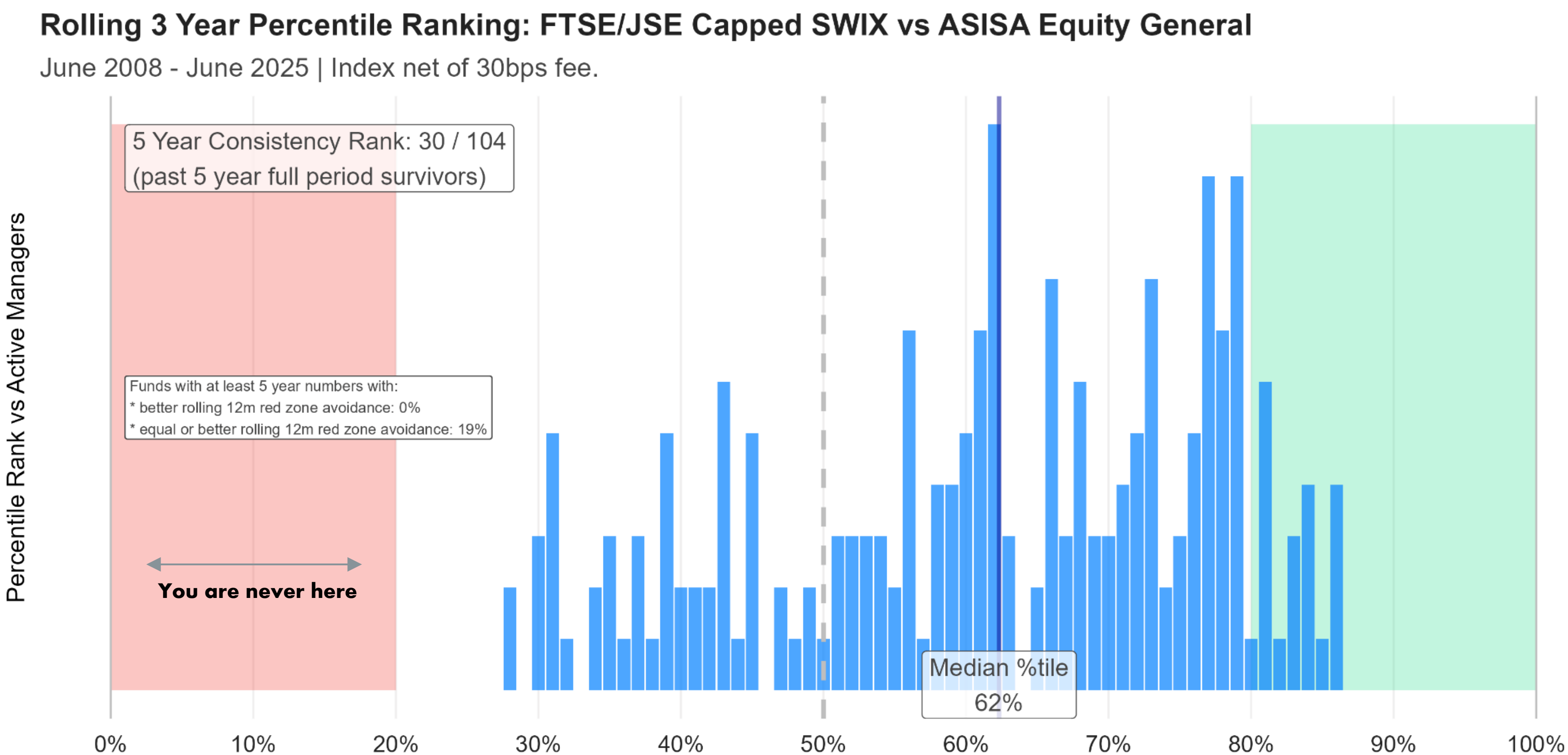
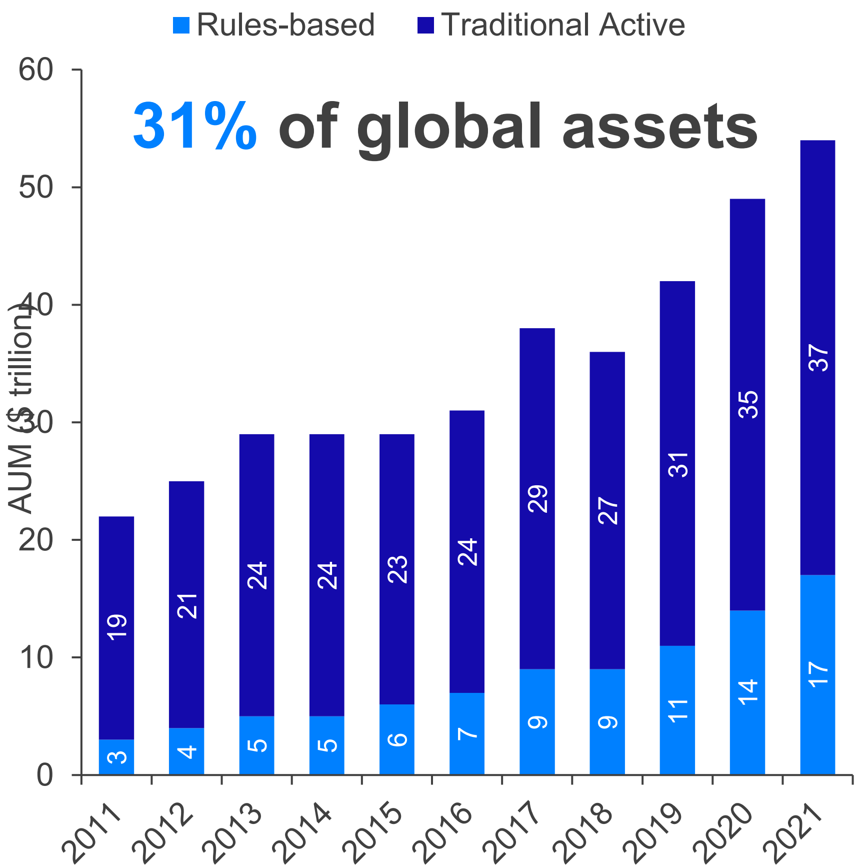
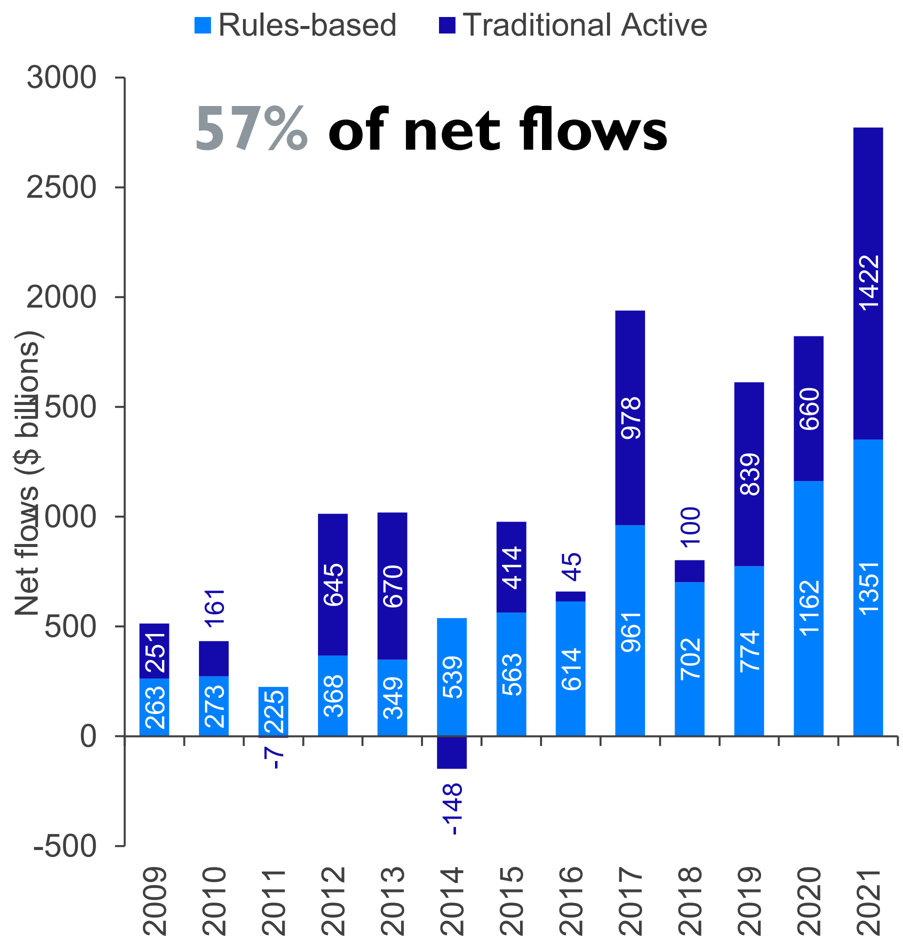


Source: Satrix. Data: Morningstar, FTSE/JSE

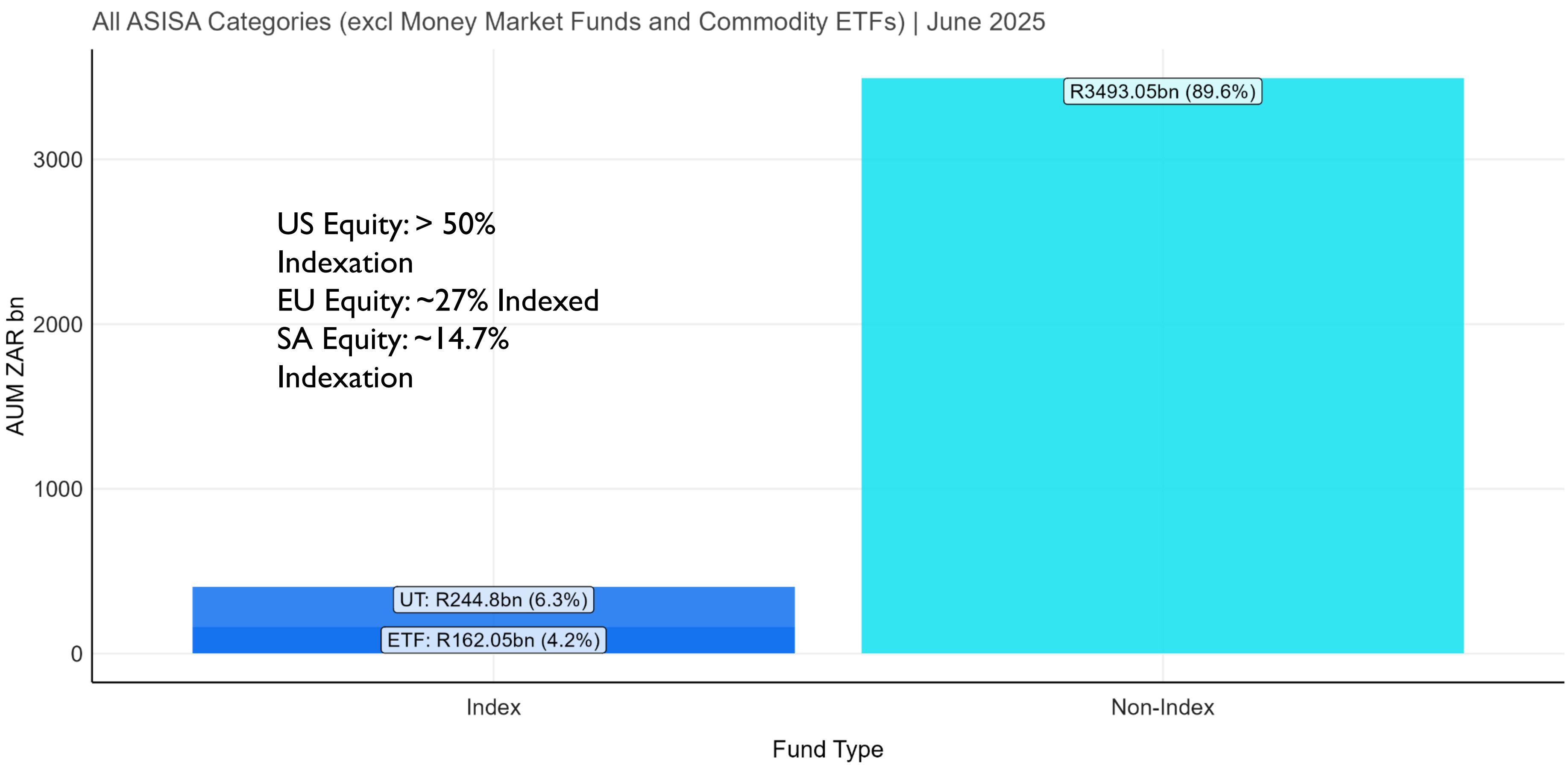
Indexation uptake: Global





Source: Morningstar & Satrix, 30 June 2023

Indexation uptake: Local

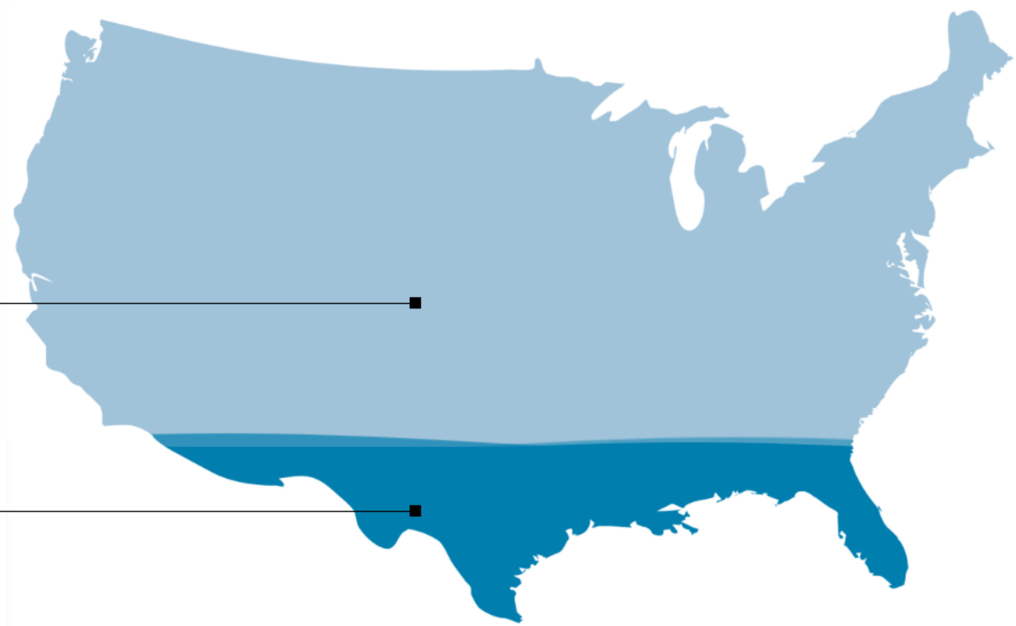


Source: Morningstar & Satrix, 30 June 2025

Are SA managers untalented? No…

United States

Percentage of All Large-Cap funds that underperformed the S&P 500®



74.27% of funds underperformed the S&P 500®

25.73% of funds outperformed the S&P 500®

1 YEAR | **3 YEARS** | 5 YEARS | 10 YEARS | 15 YEARS

Data as of Dec 31, 2022

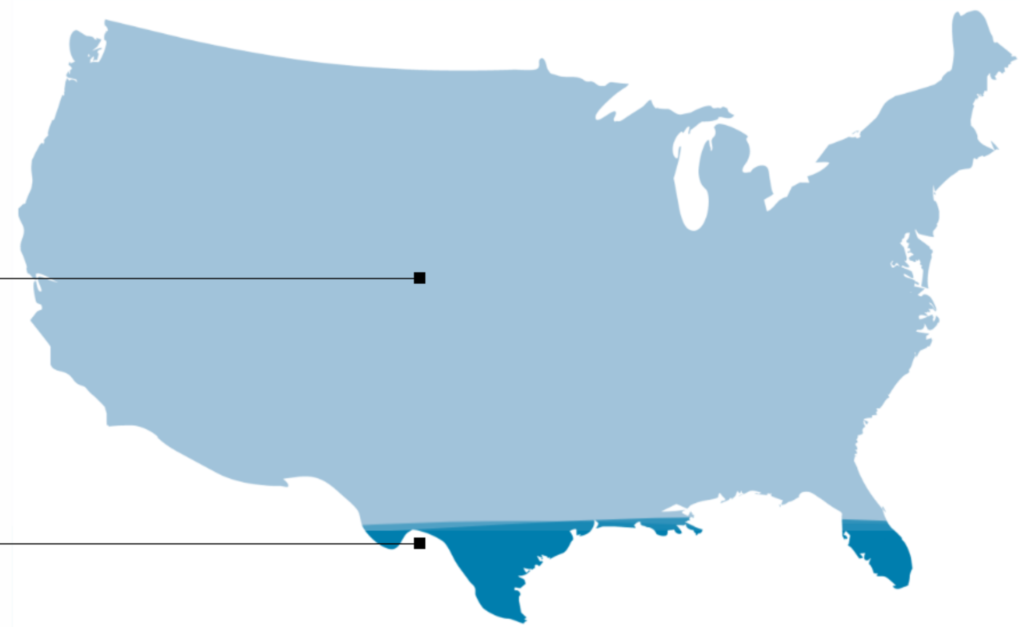
Are SA managers untalented? No…

United States

Percentage of All Large-Cap funds that underperformed the S&P 500®

91.41% of funds underperformed the S&P 500®

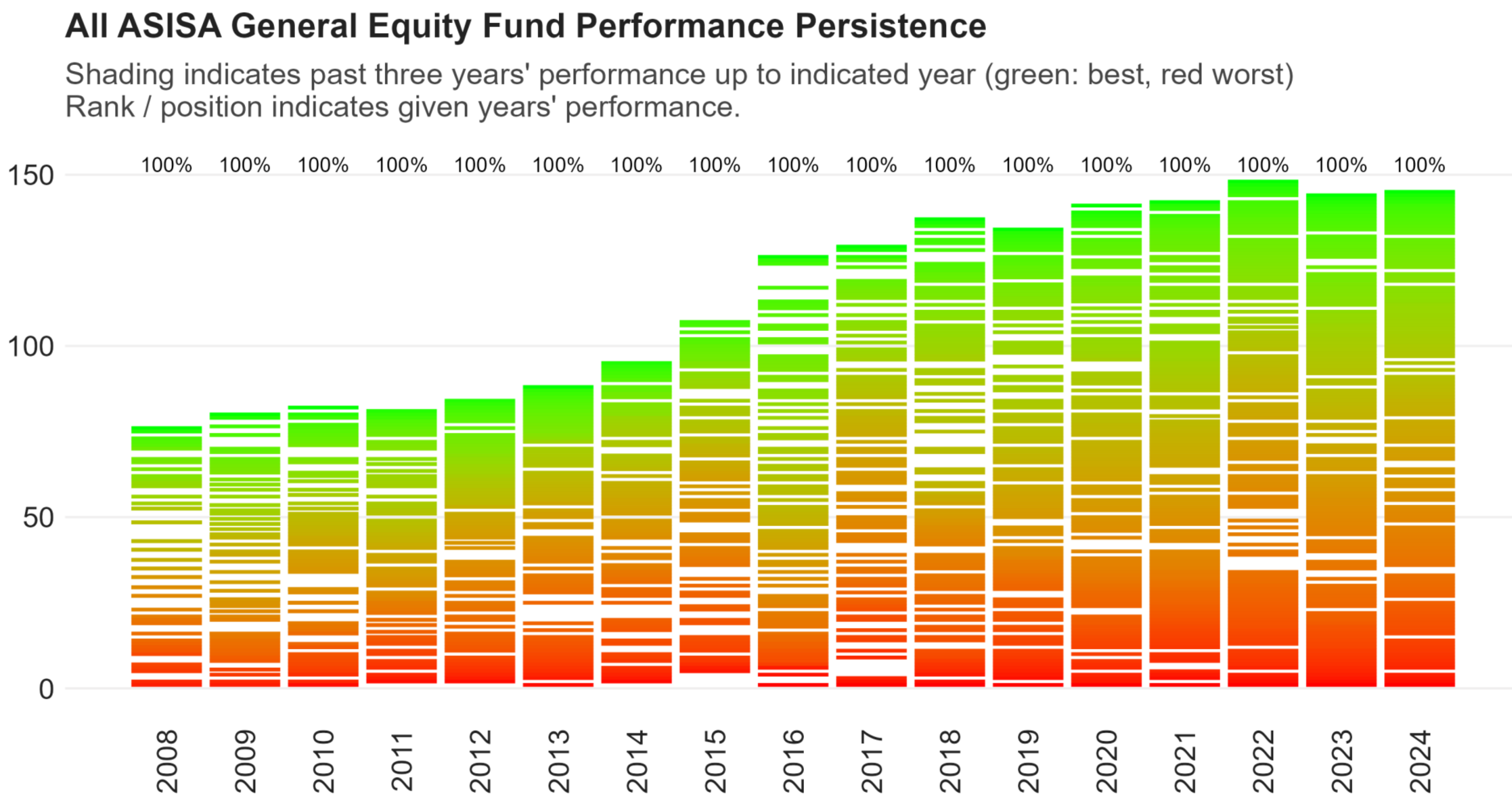
8.59% of funds outperformed the S&P 500®



1 YEAR | 3 YEARS | 5 YEARS | **10 YEARS** | 15 YEARS

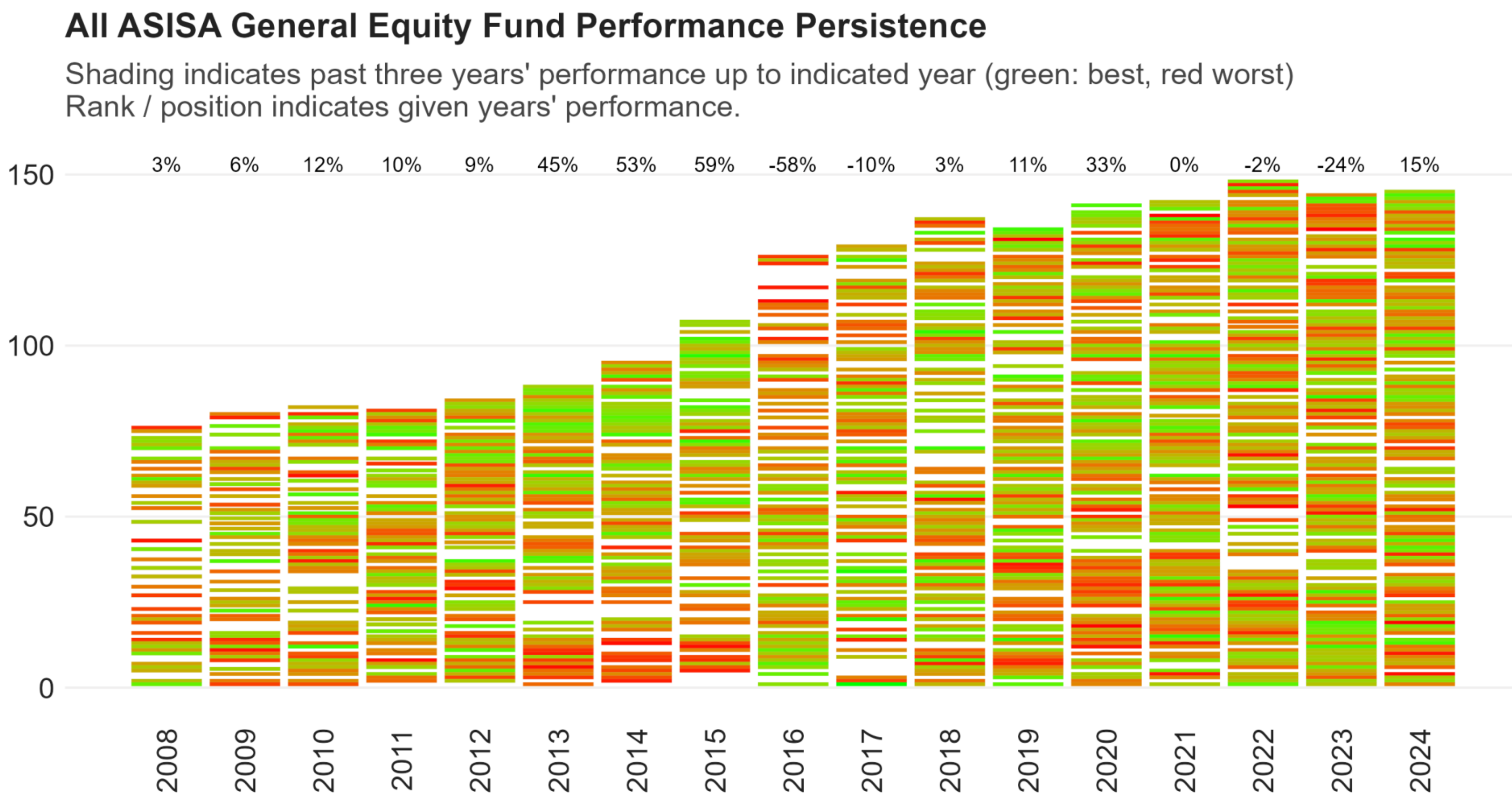
Data as of Dec 31, 2022

Simple… just find the best ones?



Source: Satrix. Data: Morningstar

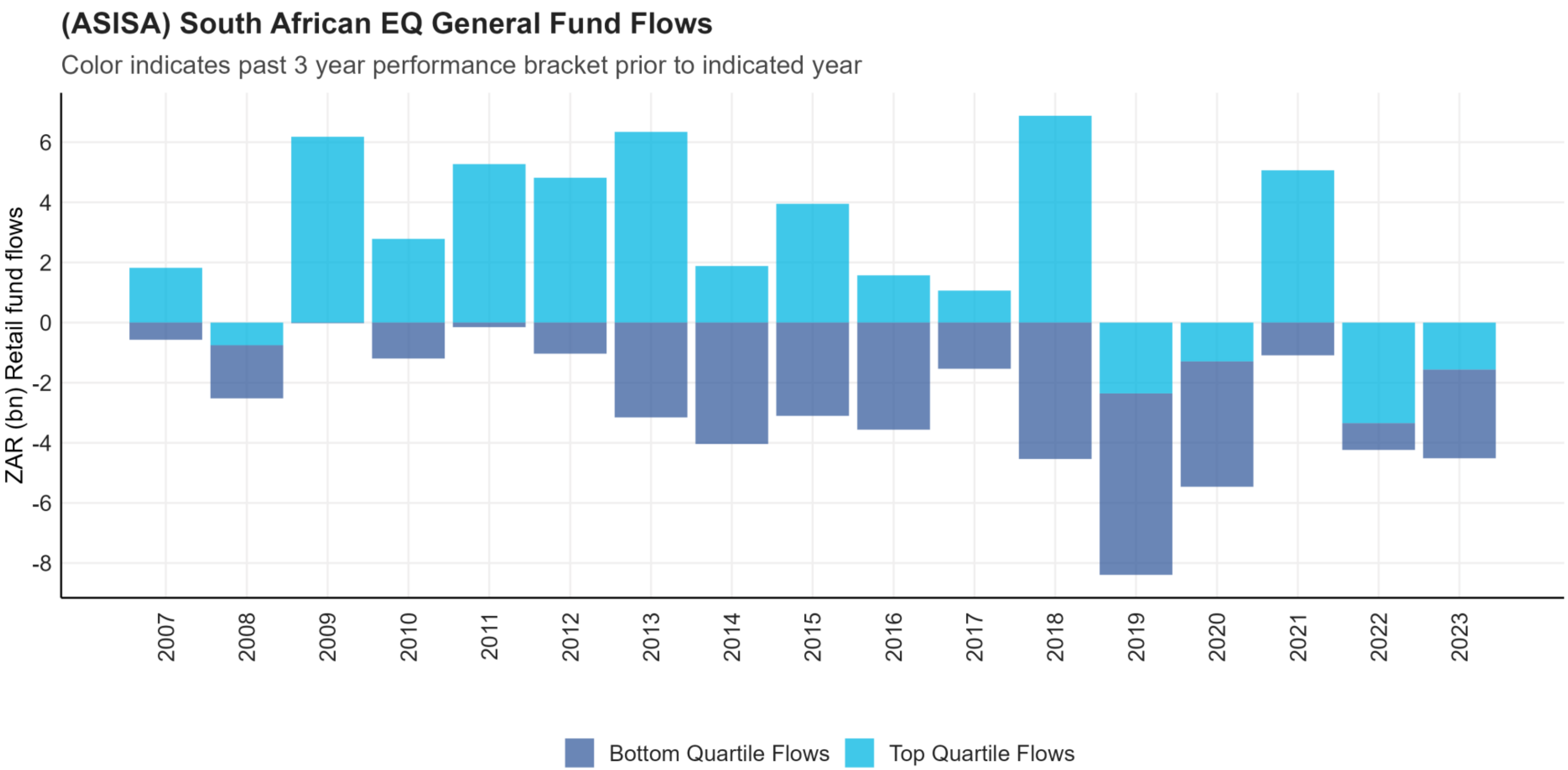
Simple… just find the best ones?



Source: Satrix. Data: Morningstar

20 Year Performance consistency

Despite evidence i.t.o. lack of persistence in performance…



Source: Satrix. Data: Morningstar

What are you paying for?

Indexation

- Investors pay for Beta

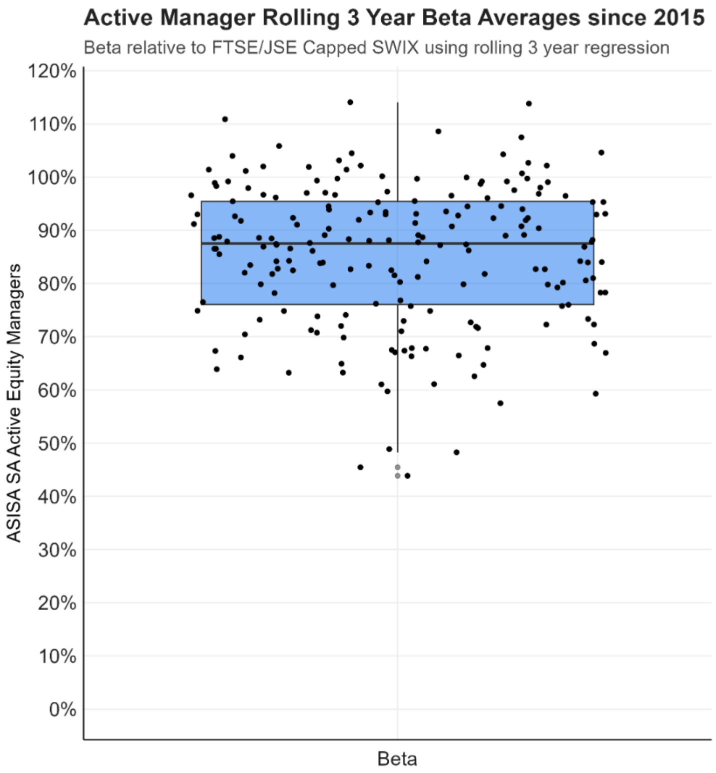
But get on average 90% Beta

(Ultimately paying a high price for Beta)

Investors should ideally:
Pay low fee for Beta
Find and pay for true Alpha

Long-Only Active

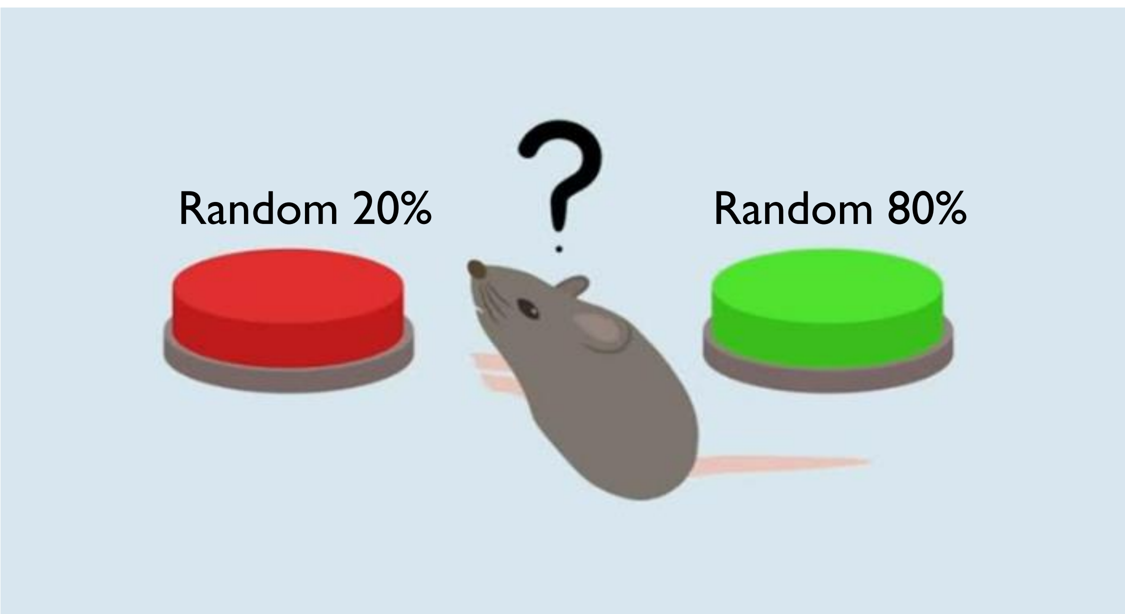
- Investors pay for Beta and Alpha



Source: Satrix. Data: Morningstar, FTSE/JSE

Why are winners chased?

- Humans are predisposed to seek order in disorder and make sense of noise.
- While stock returns resemble noise, we believe there are easy arbitrage opportunities available – you just need to pick the right expert…
  - We then believe these experts make themselves known through performance. But is this valid?



Study by Paul Zweig: "Money and Your Brain". Image source: Veritasium.

If the wrong button is hit, you get a ***shock***.
If the right button is hit, you get a ***reward***.

Best long-run strategy: only hit green

- Mice tend to hit only green
- Humans try to *find patterns* and sometimes choose red

- Mice on average get **80%** rewards
- Humans on average get **68%** rewards

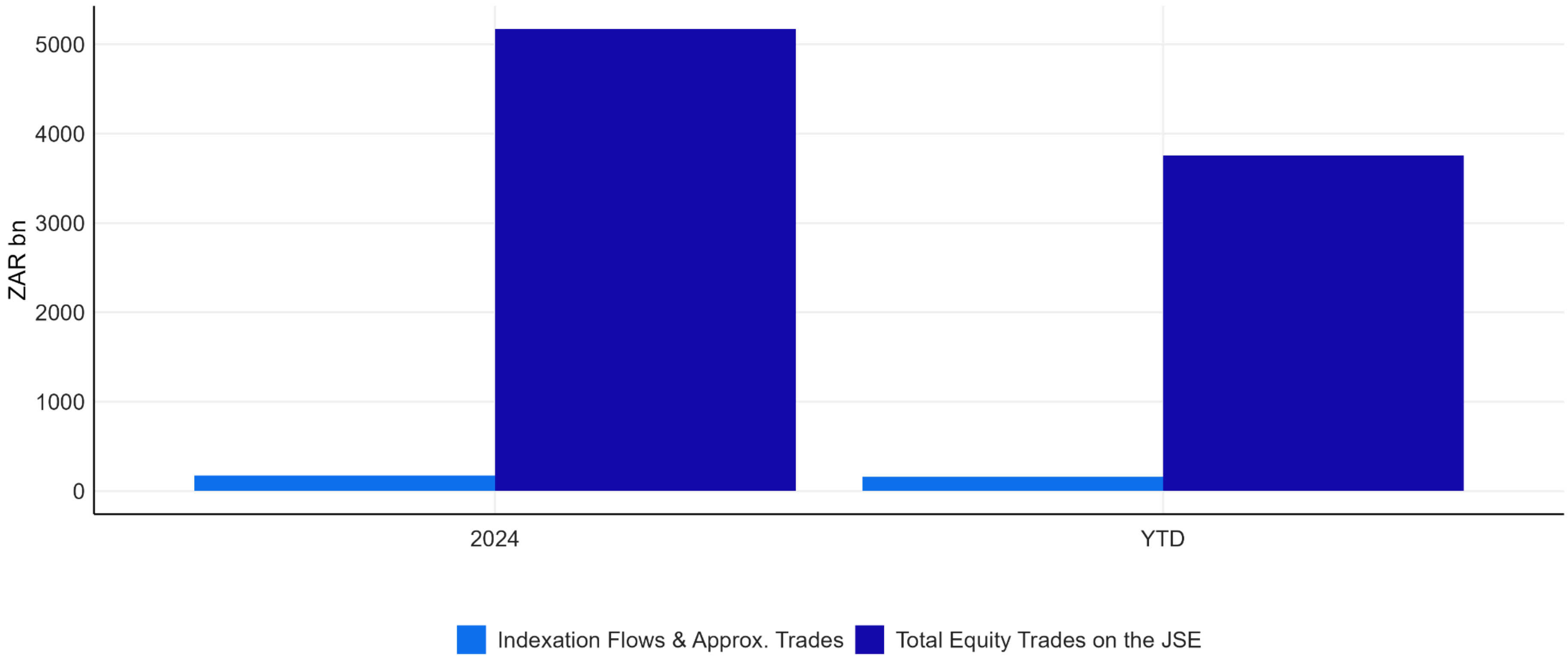
See article on behavioural factors influencing investment decisions here

What drives distortions in prices?

Does "indexation bubbles" make sense?

Total Local Equity Trade vs Total Flows and Estimated Rebalance Impact of Index strategies (UT and ETF)

Indexation Flows & Approx. Trades compared to total JSE trades. 2024: 3.3% | YTD: 4.2%



Source: Satrix. Data: JSE. Date: June 2025
For Multi-Asset strategies, we conservatively estimate for % of holdings and flows in local equities.

MPT: Assumptions – Normality of returns

- Assuming investors only care about the first two moments **(μ , σ)**, requires us to assume that asset prices behave **Normally**.
  - $E(R)$ → describes the expected value (mean) around which the probability function of the returns to holding an asset is centred
  - σ → Describes the width and shape of the distribution of all possible returns around the mean ($\sigma > 0 = uncertainty$)

<u>We can now define a term that we loosely defined earlier:</u> **RISK**

- The extent to which returns may vary from the expectation
- More formally, risk can be thought of in the MPT framework as the probability that the **Actual** return in the future differs from the **Expected** return when holding a particular asset. The degree is measured here by the asset's past variance in returns.

MPT : Mean-Variance

- Markowitzian theory therefore assumes that investors **only care** about a security's **Risk / Return** (μ; σ) profile.
- **No variance** implies a safe, secure and completely predictable outcome.
  - **One such example is a government bond (assuming bonds cannot default)**
- **Large variance** implies the potential for: **greater returns** (High-Risk High-Reward principle) but also for **greater losses** (I-knew-I-shouldn't-have-bought-it principle)…
- Markowitz assumed that investors are **risk averse** – therefore if they had the choice of holding two identical assets with the same expected return – they would choose the one with the least expected potential to deviate from this mean (i.e. choose the lowest variance asset…)

But first… A few definitions in this context

- **Asset**: for this session it will mostly be a financial asset, such as a bond or shares in a firm.
- **Portfolio**: a basket of such assets
- **Returns**: What the investor earns by holding an asset (capital gains/dividends or interest payments)
- Returns are in turn compared relative to **benchmarks**. The risk then would be underperformance of the benchmark, which include e.g: [R_B = benchmark return]
  - $\boldsymbol{R_B = 0}$ → with risk being if the return to the asset / portfolio is negative
  - $\boldsymbol{R_B = \pi}$ → With the risk being that the portfolio's return would not outperform inflation (and the investor loses purchasing power over time).
  - $\boldsymbol{R_B = R_M}$ → With risk being that the portfolio underperforms the **market** (for similar funds, or for the fund index). This could also be **sectors** or subsectors.

Diversification

- Consider a portfolio having two assets A & B:
  - With weights: (α) of A and $(1-\alpha)$ of B

The **Expected Return** for the portfolio would then be:

$$E(R_P) = \alpha . R_A + (1-\alpha)R_B$$

The **Variance** of the portfolio would then be:

$$\sigma_P^2 = Var[\alpha . A + (1-\alpha)B]$$

$$\sigma_P^2 = \alpha^2(\sigma_A^2) + (1-\alpha)^2(\sigma_B^2) + 2\alpha .(1-\alpha)\ \sigma_{A.B}$$

Where:

	R_A	= Expected Return of asset A
	σ_A^2	= Expected Variance of asset A (or B)
	$\sigma_{A.B}$	= Covariance between asset A & B

Diversification

- Remember the definition of Correlation:

$$\boldsymbol{\rho_{A.B} = \frac{\sigma_{A.B}}{\sigma_{A\bullet}\ \sigma_B}} * \quad \&: \quad \boldsymbol{-1 \leq \rho_{A.B} \leq 1}$$

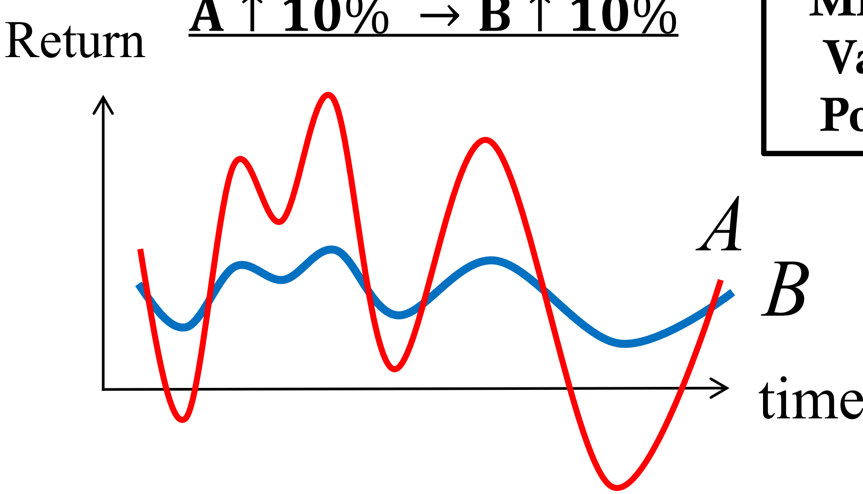
- Consider now the following scenarios:
- 1.) A & B are two risky securities [thus $\sigma^2{}_A > 0$ & $\sigma^2{}_B > 0$] and they are **perfectly positively** correlated [$\rho_{A.B} = 1$]
- 2.) A & B are two risky securities and they are **perfectly negatively** correlated [$\rho_{A.B} = -1$]
- 3.) A & B are **imperfectly** correlated risky securities [$-1 < \rho_{A.B} < 1$]
- We then graph a **mean-variance frontier** (**MVF**) to visually understand the benefits of diversification.

MVF of portfolio 1:

<u>Portfolio 1:</u> $\boldsymbol{\rho_{A.B} = 1}$

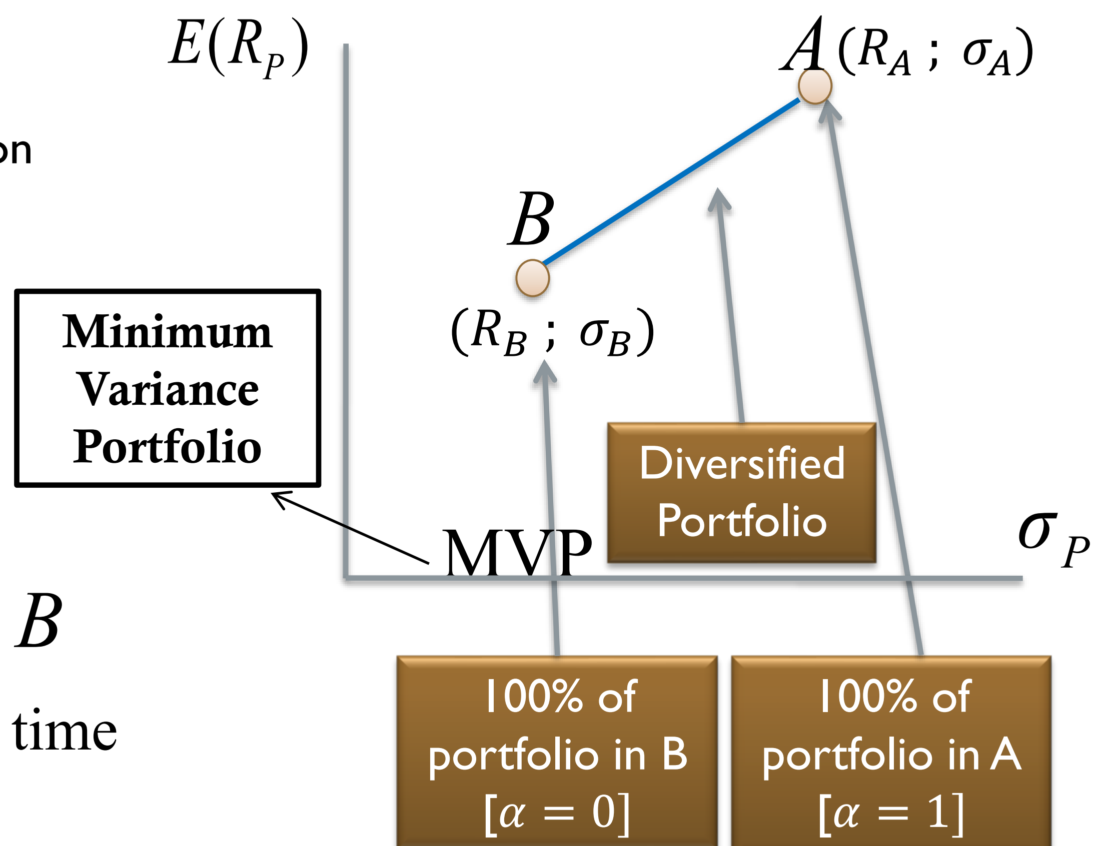
Suppose this perfectly positive correlation is deduced by looking at past returns of A & B – also assume this trend should hold for the foreseeable future – this correlation implies that :

<u>**A ↑ 10% → B ↑ 10%**</u>



Then holding A & B in a portfolio would look as follows if we graph it in a **Mean-Variance Frontier** (**MVF**):

MVF :Portfolio Risk and Returns



MVF of portfolio 1

- The **slope** of the **MVF** in this example can be calculated as (you can check for yourself…) :

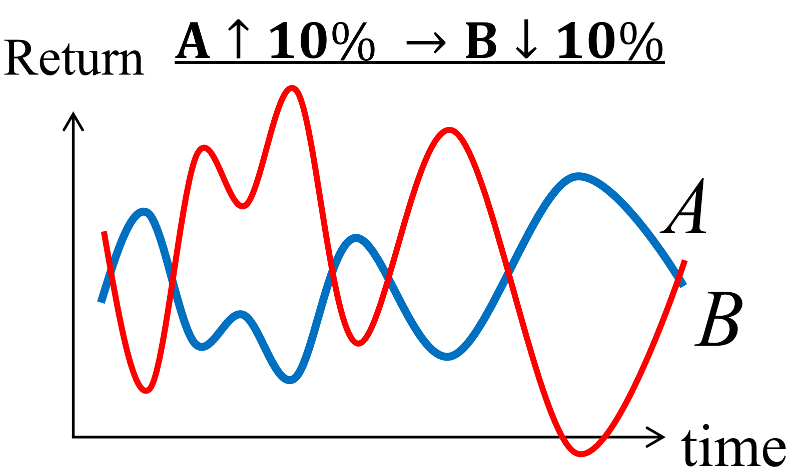
$$\frac{\Delta(\mu_P)}{\Delta(\sigma_P)} = \frac{\frac{\partial(R_P)}{\partial\alpha}}{\frac{\partial(\sigma_P)}{\partial\alpha}} = (R_A - R_B)/(\sigma_A - \sigma_B) \rightarrow \boldsymbol{constant}$$

- With the slope being constant, it implies that investors **cannot use a combination of these assets in order to diversify their portfolio.**
- A 10% loss (gain) to A would also lead to a 10% loss (gain) to holding asset B… Thus the **Minimum Portfolio Variance** (**MVP**) when only having these two variables, would be at: $\boldsymbol{\sigma_B}$ [see graph]

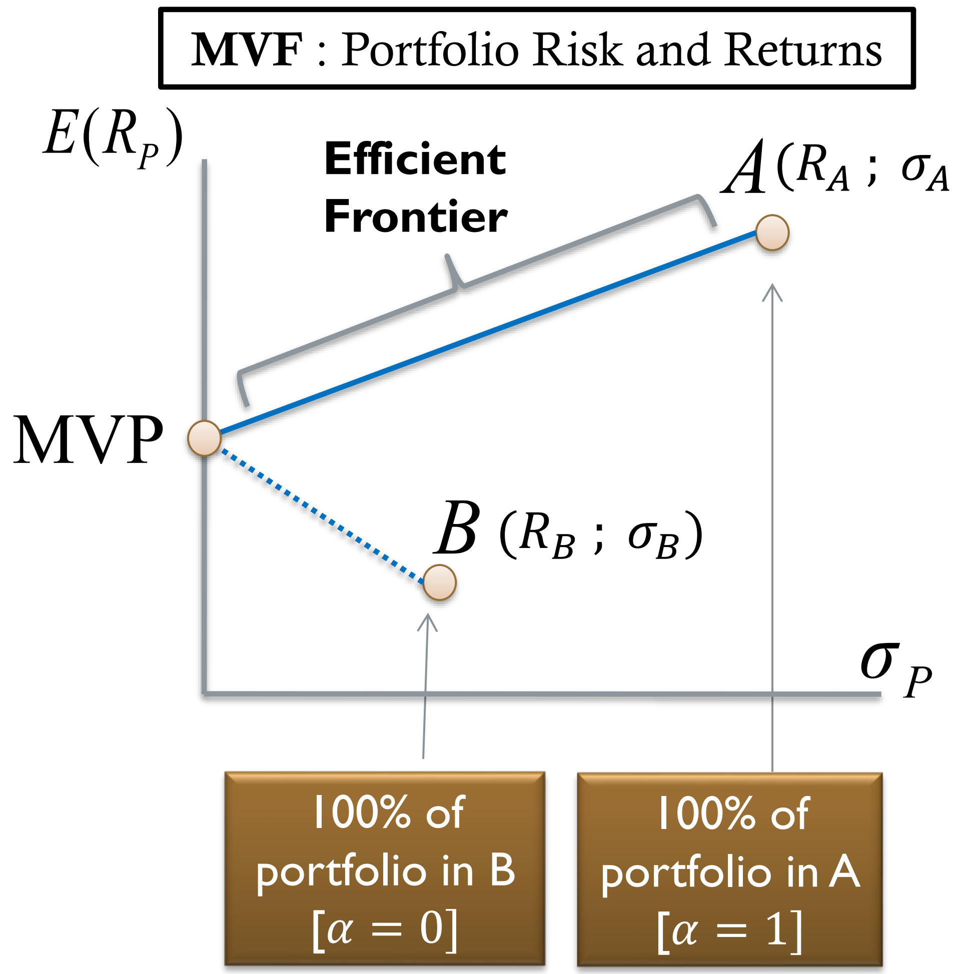
MVF of portfolio 2:

Portfolio 2: $\rho_{A.B} = -1$

Suppose this perfectly negative correlation is deduced by looking at past returns of A & B – also assume this trend should hold for the foreseeable future – this correlation implies that :



Then holding A & B in a portfolio would look as follows:



MVF of portfolio 2:

- Notice from the previous slide that combining the two assets in this case – we can **completely remove** volatility from the portfolio using a combination of both.

- Note that now a **certain combination exists** (say holding $\hat{\alpha}$ of A & $(1 - \hat{\alpha})$ of B) that would imply the **MVP** is zero… Thus: $\boldsymbol{\sigma_P = 0}$

Thus we can have **perfect diversification** (i.e. all volatility removed) in this case by holding a specific combination of both. This can then be thought of as a **risk-free portfolio** being created

MVF of portfolio 2: Formally

$$\frac{\Delta(\mu_P)}{\Delta(\sigma_P)} = \frac{\frac{\partial(R_P)}{\partial\alpha}}{\frac{\partial(\sigma_P)}{\partial\alpha}} = (R_A - R_B)/(\sigma_A - \sigma_B) > 0 \text{ if } \alpha > \hat{\alpha}$$

$$\frac{\Delta(\mu_P)}{\Delta(\sigma_P)} = \frac{\frac{\partial(R_P)}{\partial\alpha}}{\frac{\partial(\sigma_P)}{\partial\alpha}} = (R_A - R_B)/(\sigma_A - \sigma_B) < 0 \text{ if } \alpha < \hat{\alpha}$$

Notice that this implies the MVF slope has a **kink** at the combination of assets : $\boldsymbol{\alpha = \hat{\alpha}}$, where the slope goes from positive to negative… so that at this combo we have our **MVP**.

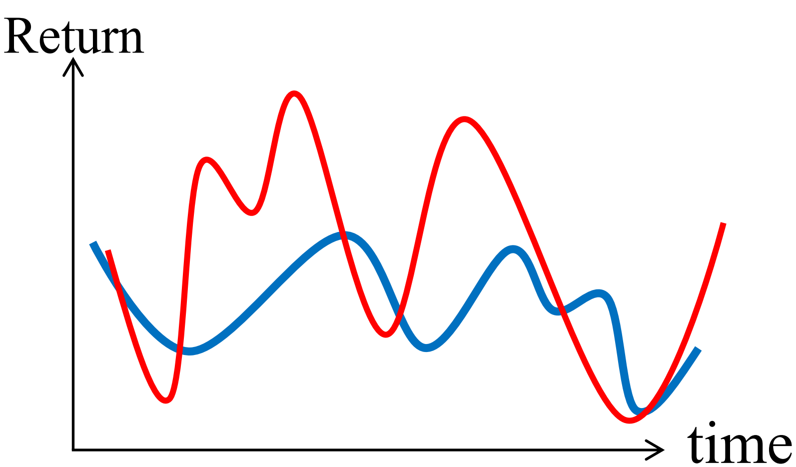
The part where the slope is positive we call the **Efficient Frontier** [because any combination under this, where $\alpha < \hat{\alpha}$, investors are not investing in an **efficient portfolio**… assuming they are risk averse…

<u>**Can you see why?**</u>

MVF of portfolio 3: Imperfect correlation

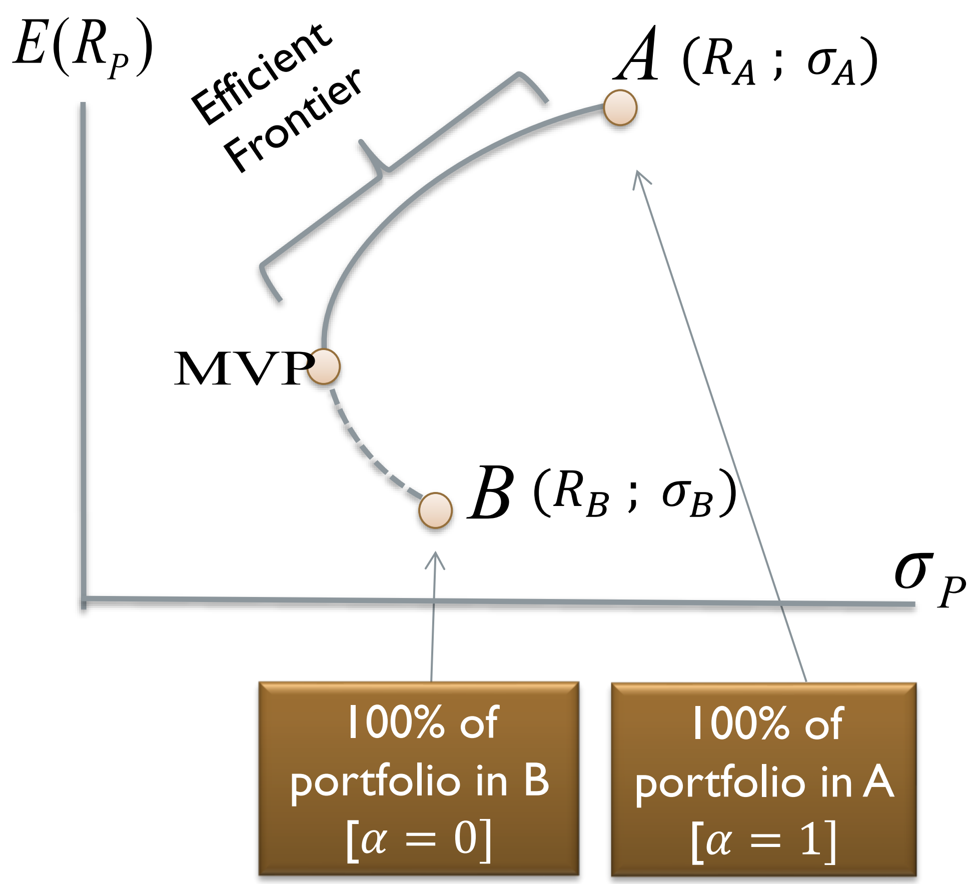
<u>Portfolio 3</u> $-1 < \rho_{A.B} < 1$

Suppose this **imperfect** correlation is deduced by looking at past returns of A & B – also assume this trend should hold for the foreseeable future – and that the co-movement of A & B remain **not perfect**.



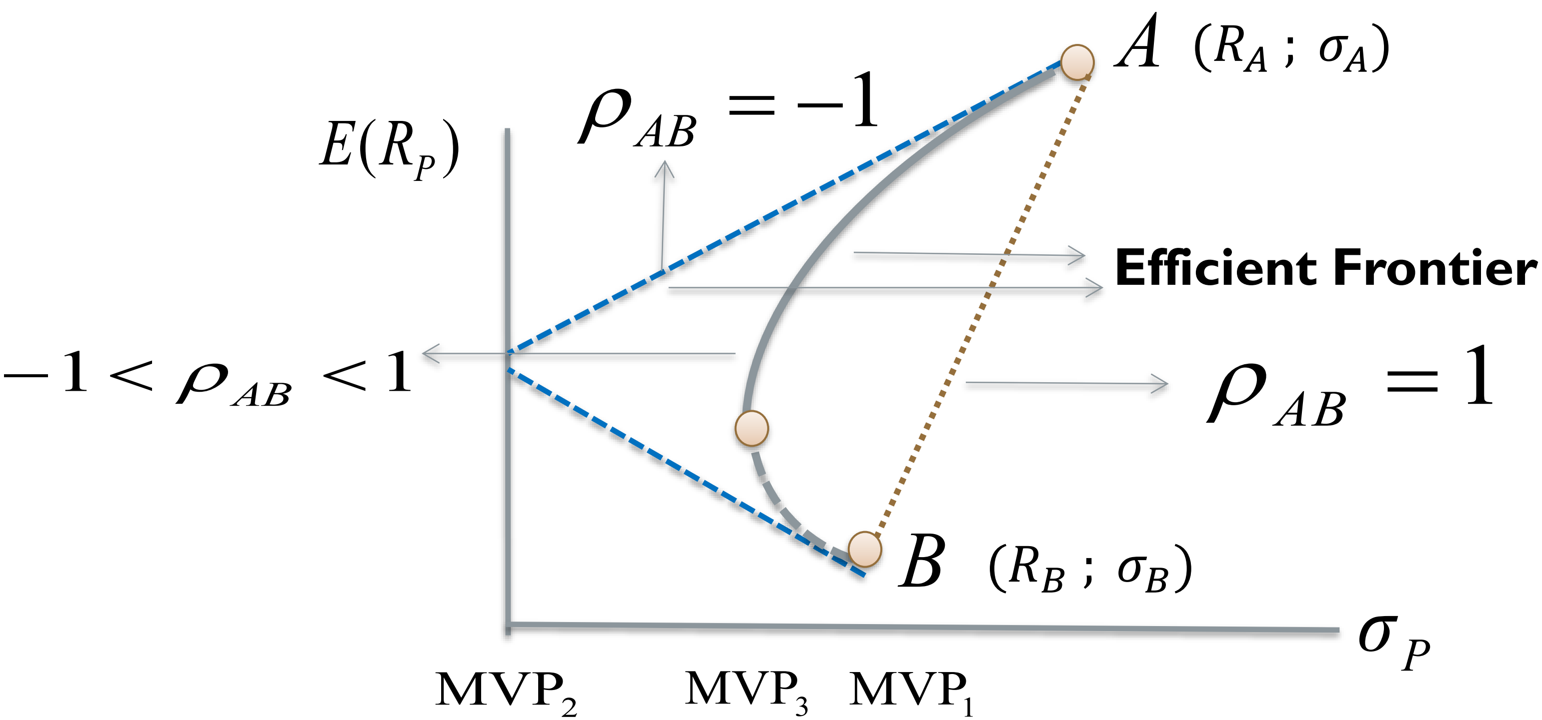
Then holding A & B in a portfolio would look as follows:

MVF : Portfolio Risk and Returns



Combining the MVFs

- Combining all 3 portfolio types' MVFs, we can see that the closer the correlation of assets in the portfolio is to perfectly negative – the greater the ability to diversify (the converse applies too).



Finding the MVP of a portfolio

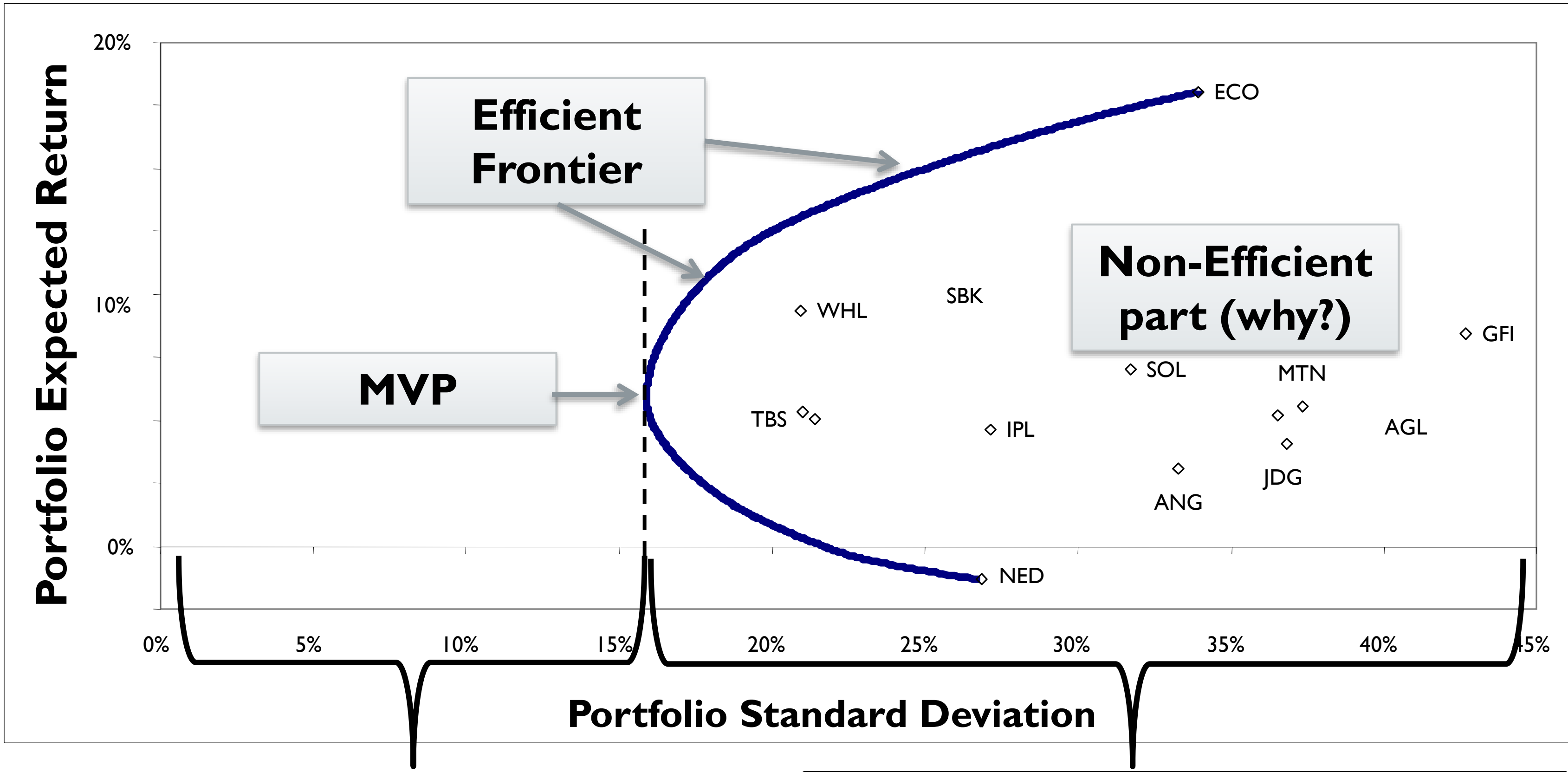
- We can use this framework to find the optimal combination of asset-weighting [$\hat{\alpha}$] that would lead to the sought after **portfolio MVP**.
- In the two asset example, we can use the following derivative to find the optimal weight:

$$\frac{\partial \sigma_p^2}{\partial a} = 2a\sigma_A^2 - 2\sigma_B^2 + 2a\sigma_B^2 + 2\sigma_{AB} - 4a\sigma_{AB} = 0$$

$$\hat{a} = \frac{\sigma_B^2 - \sigma_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}}$$

- The math becomes a bit more complicated if we increase asset size though…
  - Notice that **finding such a MVP we need to assume the historic trends should persist** – i.e. that past correlations will persist into the future!

MVF for many risky securities



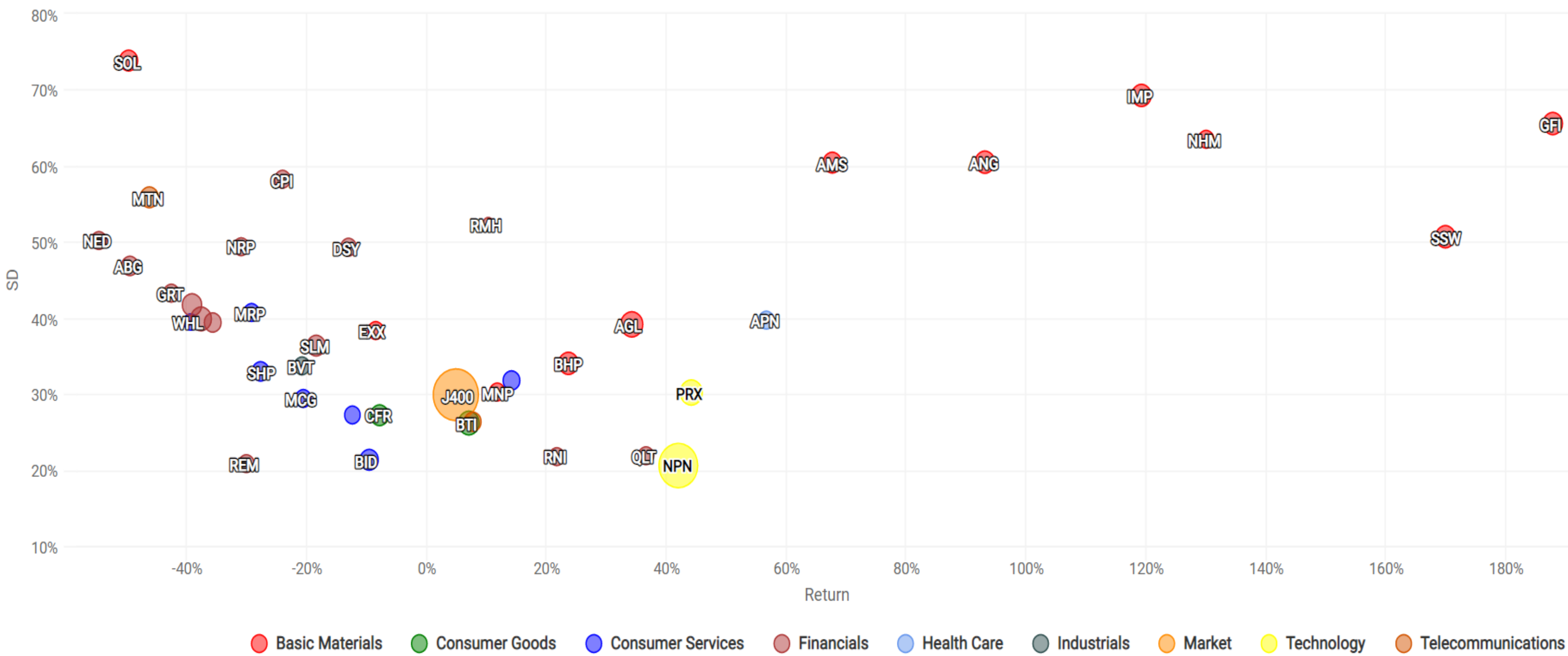
Non-Diversifiable (market) Risk: Market specific risk that cannot be removed through diversification as all assets are exposed to it.	**Diversifiable (Idiosyncratic) Risk:** Asset specific risk that can be controlled for through diversification

Non-Diversifiable (market) Risk:
Market specific risk that cannot be removed through diversification as all assets are exposed to it.

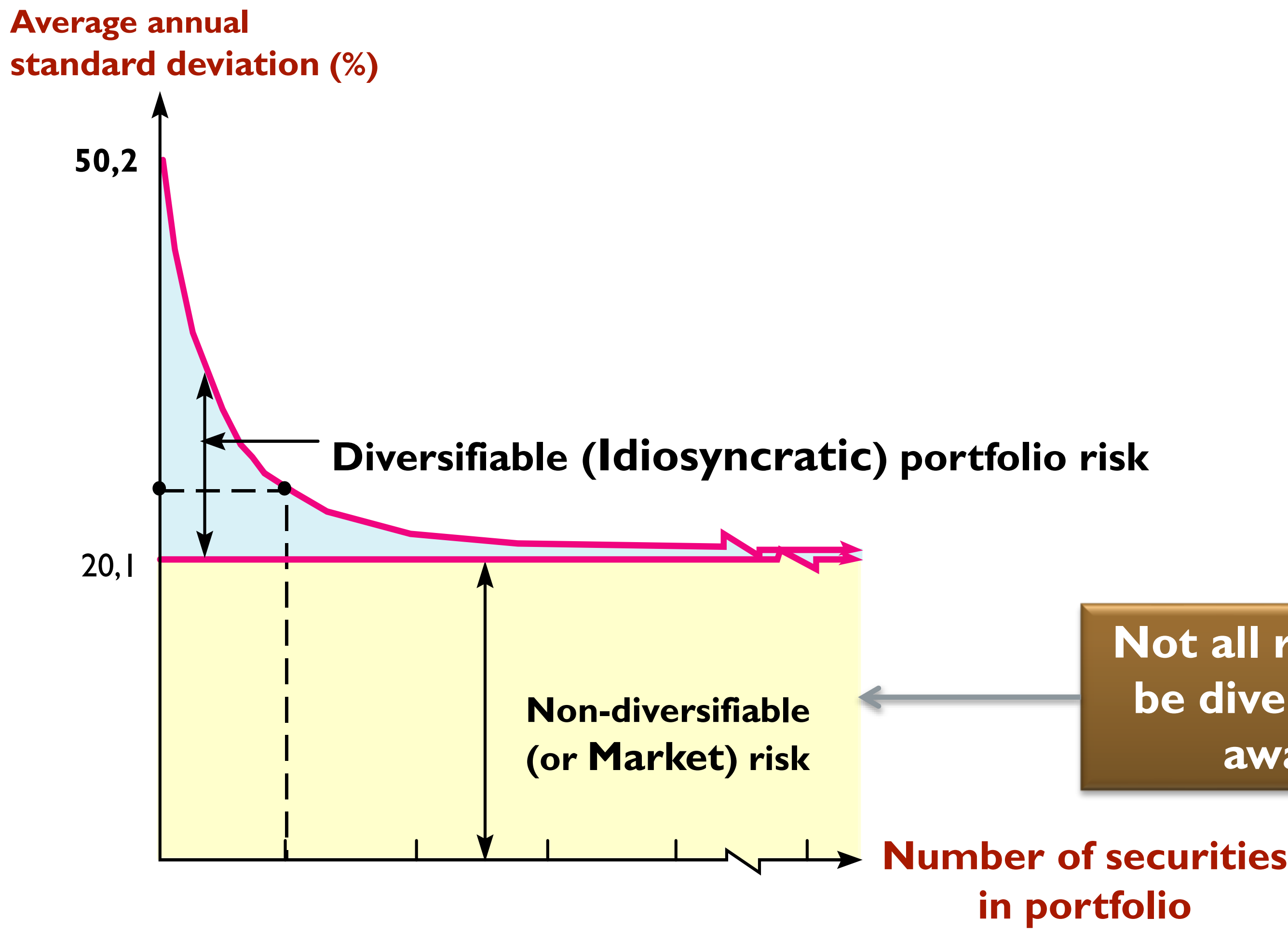
Diversifiable (Idiosyncratic) Risk:
Asset specific risk that can be controlled for through diversification

SWIX mean-variance (6 Aug 2020)

DTop: Mean-SD - Last 12 Months



The effect of increasing the number of (non-perfectly correlated) risky securities in your portfolio



Not all risk can be diversified away

Sharpe Ratio : Including the Risk-free asset

- Right, so we have now seen that **diversification** could lead to a **lowered risk** for a portfolio → NB: **by** combining non-perfectly positively correlated assets.
- But how do we actually **use** this in **pricing** assets? As mentioned before, the ***all-eggs-in-one-basket*** analogy has been with us for 100s of years…
- But finding a fair price based upon an asset's risk **relative to the market's measure of risk** – ***that*** is a novel concept.
- If you look at the last slide – pricing risk would essentially be a **subjective exercise**, as it clearly depends on the investor's **own valuation of risk**…

  i.e. how risk should be priced has not yet been answered, and this is what we now set out to do.

  - We now need a **mutually agreed upon** price of risk in the market….

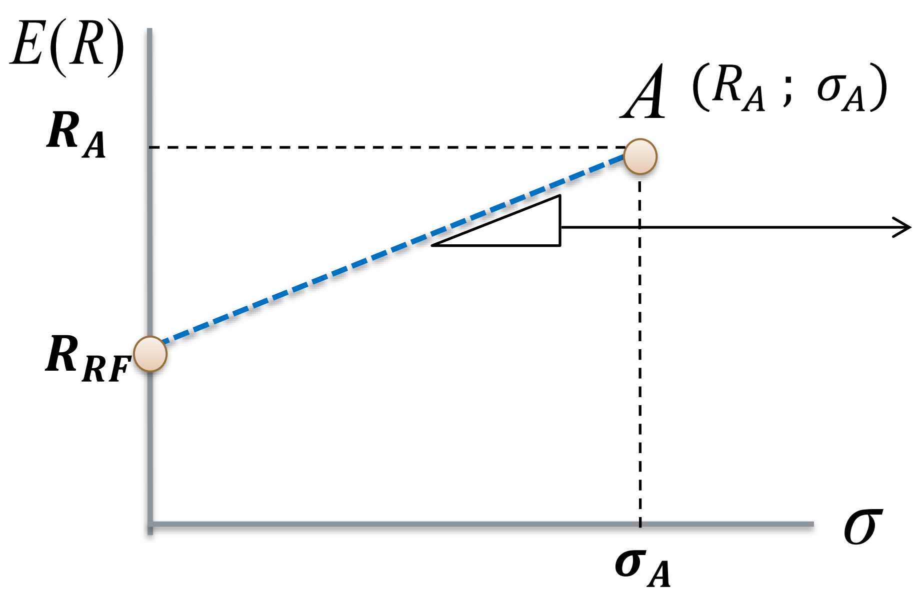
Sharpe Ratio : Including the Risk-free asset

- Suppose there exists an asset in the market that can be regarded as **risk free (RF).**

- This RF asset then has a risk / reward profile of:

$$E(R_{RF}) = \boldsymbol{R_{RF}} \;\; \& \;\; Var(R_{RF}) = \sigma_{RF} = \mathbf{0}$$

If we combine this **RF asset** with risky asset A, we find the **Sharpe Ratio,** which is an indication of asset **A's risk relative to RF**:



Slope of the line :
Sharpe Ratio

$$\frac{\boldsymbol{dR}}{\boldsymbol{d\sigma}} = \frac{\boldsymbol{R_A - R_{RF}}}{\boldsymbol{\sigma_A}}$$

Sharpe Ratio

- Note that the slope of the two assets implies the trade-off of seeking more return by taking on more risk.
- What this means is that the **Sharpe Ratio** measures the **risk-adjusted return** of asset A: being thus the **added compensation** (in terms of return) required for an investor **to hold A**.
- This now allows us to find a **market efficient price** for any asset → if we could assume a **commonly agreed upon** risk-free rate of return (such as on a short term government bond) exists*
    - *we will return to this concept later…*

Market Return

- Suppose that, in addition to the Risk-Free asset, there also exists a **Market portfolio** – which perfectly mimics the market risk / reward profile.
  - This can be thought of as a **market index**, such as the JSE ALSi or the S&P 500.
- Holding a share in such a portfolio is analogous to holding a **piece of the entire market (we'll return to indices later)**.
- Notice that, by definition, the market index (**M**) **implies an efficient portfolio** that has all **idiosyncratic** (or asset specific) risk eliminated – with only the undiversifiable **market risk** remaining… Thus it lies on the market's **Efficient Frontier by definition (as it reflects the market)!**
- The risk-reward profile of **M** looks as follows:

$$E(R) = \boldsymbol{R_M} \; ; \quad Var(R) = \boldsymbol{\sigma_M}$$

Combining the **RF**-asset and **M**

- Suppose we combine these two asset classes (**RF and M**) into one **MVF** – ***then*** we would be able to find a means of pricing risk fairly in the market…
- A portfolio that has a choice between investing some part in the market index (**M**) and some part in a risk-free asset (**RF**), would have the following **risk / reward** profile *(following the same reasoning as earlier with asset A & B)*

$$E(R) = \alpha . R_M + (1-\alpha) . R_{RF}$$

$$Var(R) = \sigma_P^2 \quad = Var[\alpha . M + (1-\alpha)RF]$$

$$\sigma_P^2 \quad = \alpha^2(\sigma^2{}_M) + (1-\alpha)^2(\sigma_{RF}^2 \quad) + 2\alpha.(1-\alpha)\ \sigma_{M,RF}$$

$$= \boldsymbol{\alpha^2(\sigma_M^2 \quad)} + 0^*$$

**Because: $\sigma_{RF} = \sigma_{M;\,RF} = 0$

Combining the **RF**-asset and **M**

- From the last slide, it is clear that the portfolio's risk is **only dependent** on the proportion of **M** in the portfolio.

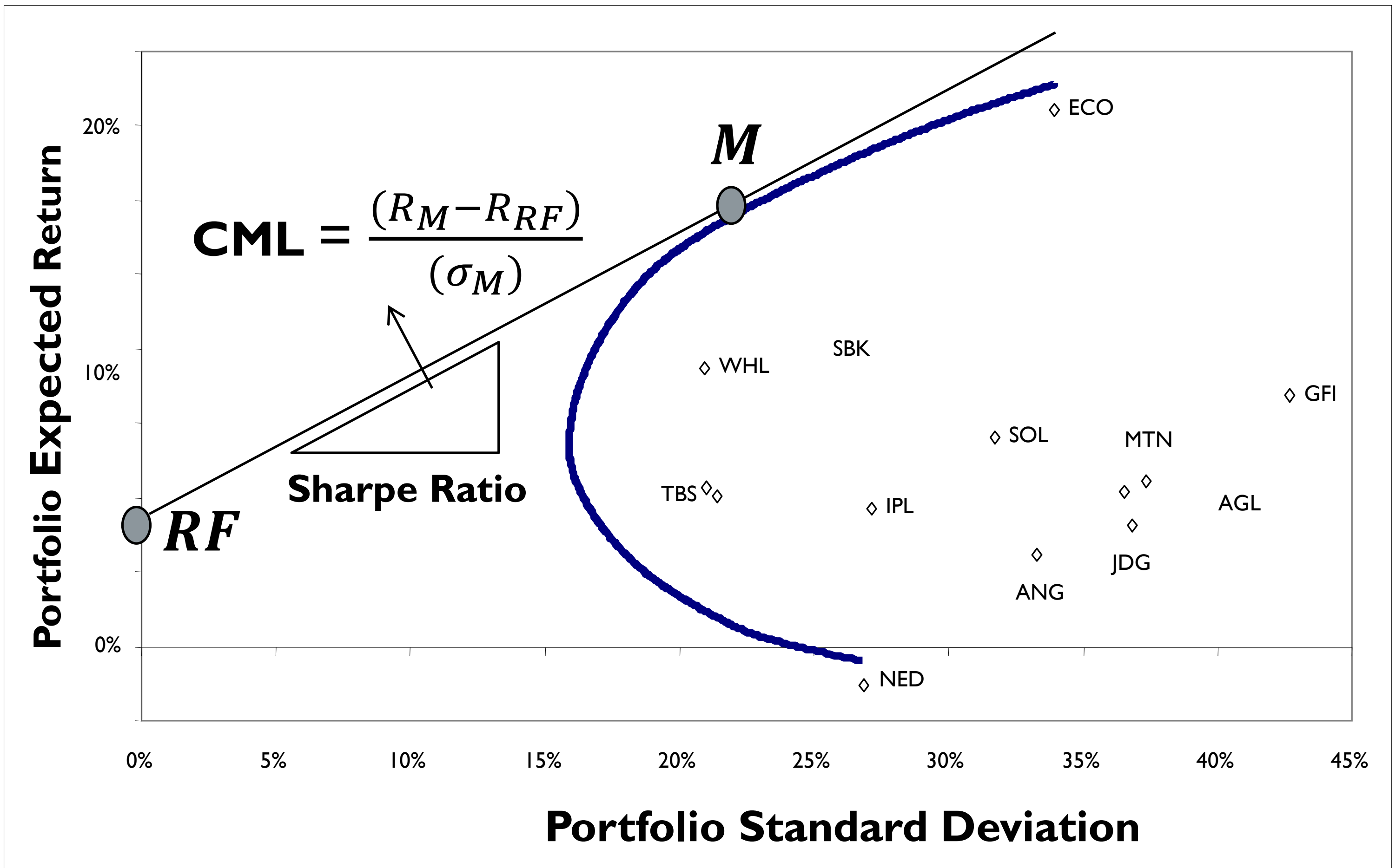
<u>We can now define the **MVF slope** as:</u>

$$\frac{\Delta(R_P)}{\Delta(\sigma_P)} = \frac{\frac{\partial(R_P)}{\partial\alpha}}{\frac{\partial(\sigma_P)}{\partial\alpha}} = \frac{(R_M - R_{RF})}{(\sigma_M)}$$

- This is known as the CML (Capital Market Line), which is **constant**
- Notice that **ALL investors** face the same **CML**, as they all face the same **RF** and **M** [e.g. the same government bond yield and market index]

Combining the **RF**-asset and **M**

- An efficient market portfolio and a risk-free asset combined on SA data
- **CML** → **Sharpe Ratio** connecting **RF** and **M**



CML

- The efficient Market portfolio (**M**) would thus be where the Sharpe Ratio line (connecting the RF and M) intersects the **E**fficient **F**rontier **once**. This is known as the **CML**.

- Check on the previous slide, that any index M that lies **inside** the Efficient Frontier would lead to a Sharpe Ratio line that intersects the EF twice.
  - This would lead to **all investors** to then optimally choose the **same risky bundle** of assets (**M**) **-** that lies on the **CML**. AND THIS IMPLIES that all investors share the **same marginal valuation for risk**, pricing it as:

$$\frac{\partial R_p / \partial a}{\partial \sigma_M / \partial a} = \frac{R_M - R_{RF}}{\sigma_M} > 0$$

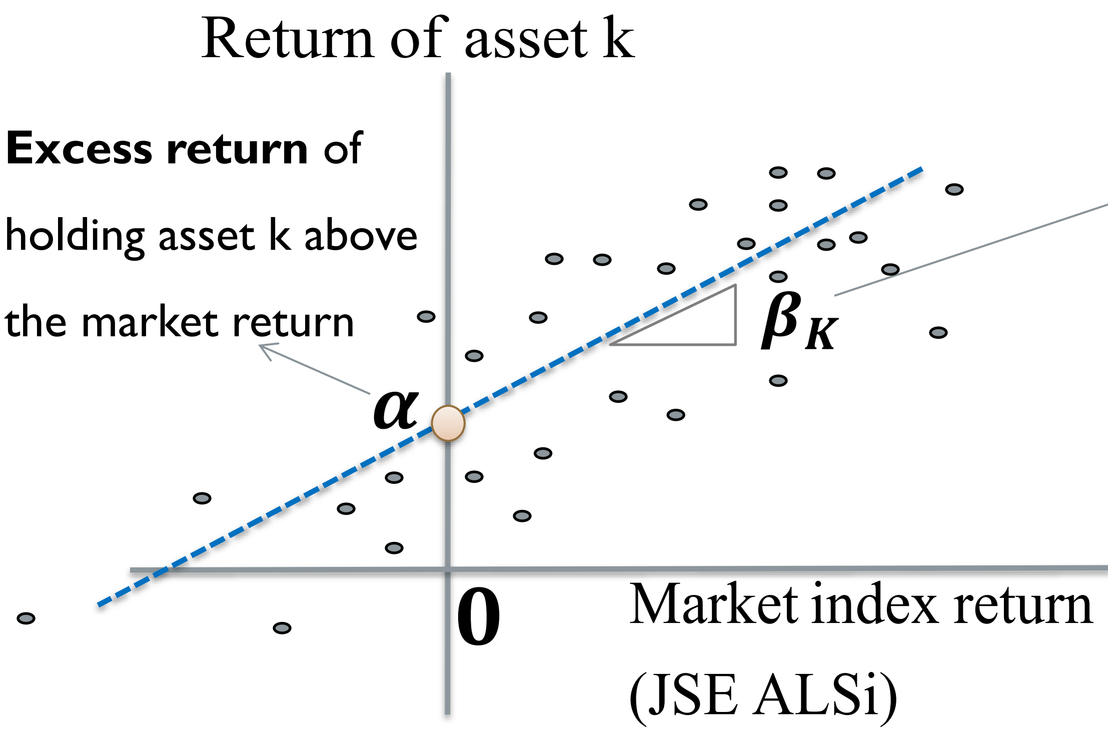
Using CML to price individual assets…

- This in turn suggests that investors should choose a portfolio that lies on the CML → i.e. they can hold any combination of M and RF in their portfolio.
- Of course, investors differ in their risk appetite, and would thus choose to hold different combos of **M** and **RF** – with the amount of M held determining the amount of risk the portfolio is exposed to.

- As all efficient portfolios ***should*** lie on this CML, we've now finally arrived at a **system-wide consensus of how to price risk**…
- We now want to use this insight to price **individual assets** to arrive at a **fair price** for assets i.t.o. its **specific risk / reward profile** relative to RF and M…

Characteristic Line of asset k

- To price individual assets in this framework, we first need to determine its unique **characteristic line** i.t.o. the **market index**…
- For this we regress the returns (say weekly) of asset k and the market index **M** to find the slope ($\boldsymbol{\beta}$) and intercept ($\boldsymbol{\alpha}$):



$$\boldsymbol{\beta_K = \frac{d(R_k)}{d(R_M)} = \frac{\sigma_{k;M}}{\sigma_M^2}}$$

Thus $\boldsymbol{\beta_K}$ is the change in return of $\boldsymbol{k}$ relative to changes in ***M***.

$\boldsymbol{\beta_K = 1}$ implies:

$$R_M \uparrow 10\% \rightarrow R_k \uparrow 10\%$$

Beta and Alpha

- This method of empirically determining an asset's characteristic line using OLS techniques is widely used in practice in an effort to see how a particular asset relates to the **benchmark (M)**
- $\boldsymbol{\alpha} =$ intercept and implies the **excess return** above (or below) the market when holding k.
- $\boldsymbol{\beta} =$ is the characteristic line's slope (fitted using OLS) and can be thought of as the ***sensitivity of a share's price relative to changes in the market price.***
- REMEMBER FROM OLS: $$\beta_X = \frac{Cov(R_k, R_M)}{Var(R_M)} = \frac{\sigma_{k,M}}{\sigma_M^2}$$

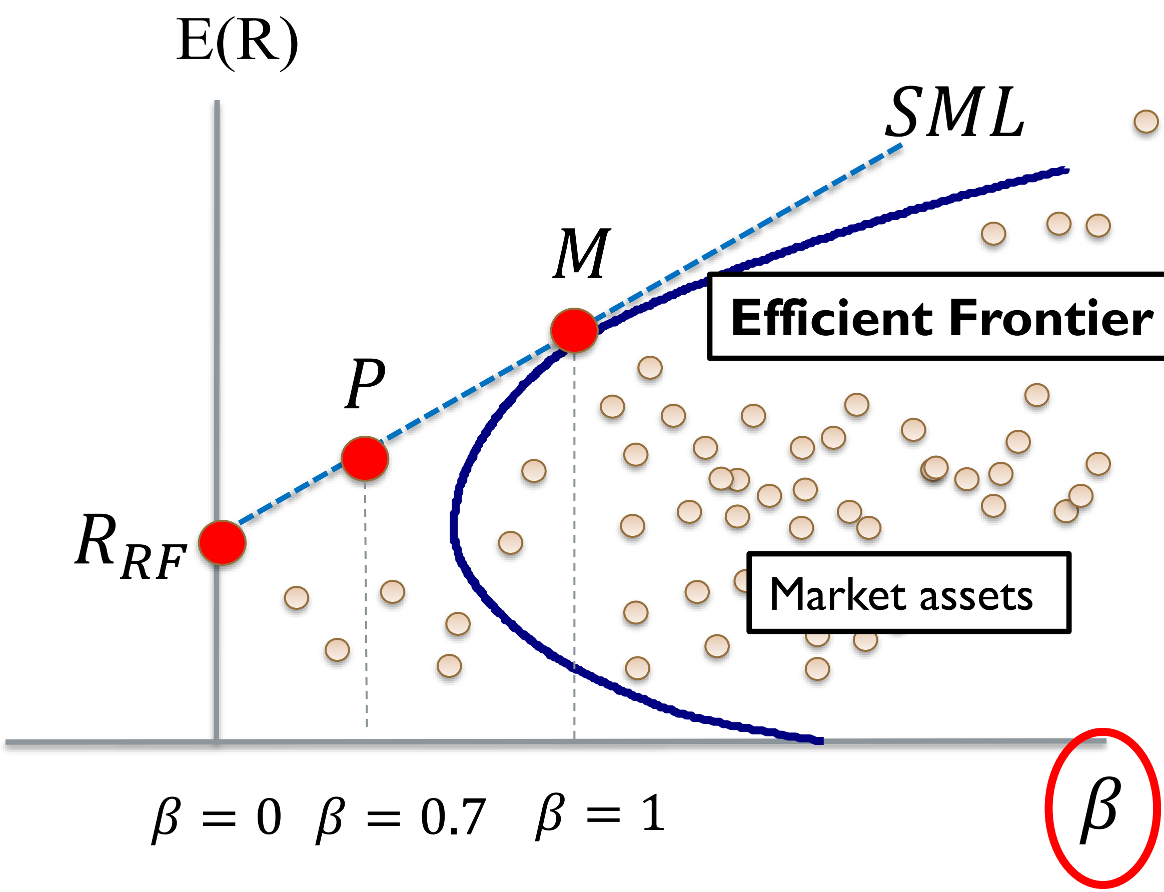
SML

- Now, combining these concepts we find a means of pricing risk…
- Notice the risk-adjusted return in this model is determined by:
  - The risk-free asset (RF) & The Market Portfolio (M)
- Any asset / portfolio's risk can then be determined relative to these two assets (RF & M).
- Let β_k then be the risk of holding asset k (i.t.o. the market portfolio) which is in market M, or similarly let β_P be the relative risk of a portfolio that combines RF and M (or other combos of assets on the SML).

- This leads to the forming of the **S**ecurities **M**arket **L**ine (With $\boldsymbol{\beta}$ now on the X-axis)

SML

- SML: represents the relationship between the $\boldsymbol{E(R)}$ of individual assets / portfolios (P) and its ***relative risk*** **-** as measured by its **sensitivity** to market movements ($\boldsymbol{\beta}$)



SML is very much like the **CML**, with the latter measuring E(R) i.t.o. **total risk** and **SML** measuring E(R) i.t.o. **relative** (to market) **risk** .

In this example

Portfolio $\boldsymbol{P}$ has $\beta = 0.7$ → implying it is similar in structure to Portfolio with **70%** $\boldsymbol{M}$ **and 30%** $\boldsymbol{RF}$

SML

- From the previous graph, it can be noted that all portfolios that lie **on the SML** is made up of **ONLY M and RF** – and are **more efficient** than the portfolios that lie on the Efficient Frontier (which is below the SML).
  - Why is this the case? Because remember that assets have idiosyncratic (asset specific) risk, that can only be removed by combining it with other assets in the market…
- The asset M, or market portfolio, thus has all **idiosyncratic risk** removed, and as such has less risk than individual assets… (see earlier slide)

Motivation for passive investments?

- Note that the SML is essentially a **theoretic motivation** for using indexed mutual fund investment (i.e. investing in JSE ALSi i.s.o. buying individual assets on the JSE)
- WHY?
  - It can be seen that **any portfolio** lying on the Efficient Frontier in this framework that is not a combination of **M** and **RF** only: is not efficient in terms of its risk/reward profile.
- In practice, although portfolio could yield a return above the Efficient Frontier (that can be thought of as ***value*** for investors, ask any fund manager and they will say they do that), EMH theory suggests that is not predicable ***ex ante*** (other wise all investors will go into the same winning fund, making it so large that it becomes M).

CAPM: Assumptions

- We can now AT LAST! use this framework to find **a** (not necessarily **the**) fair price of risk in the market, given the following assumptions hold:
  - There exists a **RF** asset & a good proxy for **M**
  - We **ignore** taxes and trading / information costs.
  - **Returns** are **linearly related to risk** (with the latter being defined in the market as ordinary return volatility, or return variance)
  - Investors are **risk averse** (if $\mu_A = \mu_B$, the investor chooses $\min(\sigma_A, \sigma_B)$
  - Expectations in the market regarding E(R) and Var(R) are **homogenous**.
  - All investors have the same time frame horizon.
  - Investors can leverage to construct portfolios past $\beta = 1$ (i.e. they can borrow at the RF rate and invest in M)
  - Returns are **normally distributed** and the **past is a good indicator** of the future movements in asset prices.
  - Information is freely and immediately available to all investors.

CAPM

- Using the **MPT** framework **and** the **SML** insights, we can now find the fair (i.e. risk adjusted) price of any asset if we know it's $\boldsymbol{\beta}$ (which we find most simply using OLS on past returns).
- Then the fair price for any asset k is such that the expected return from holding it **satisfies the CAPM equation:**

$$\boldsymbol{R_{CAPM(k)} = R_{RF} + \beta_k \,.\, (R_M - R_{RF}) + \alpha}$$

Fair (risk-adjusted) return to holding k

Risk-free return

Risk of holding asset k relative to the non-diversifiable market risk

Amount of mkt risk in k

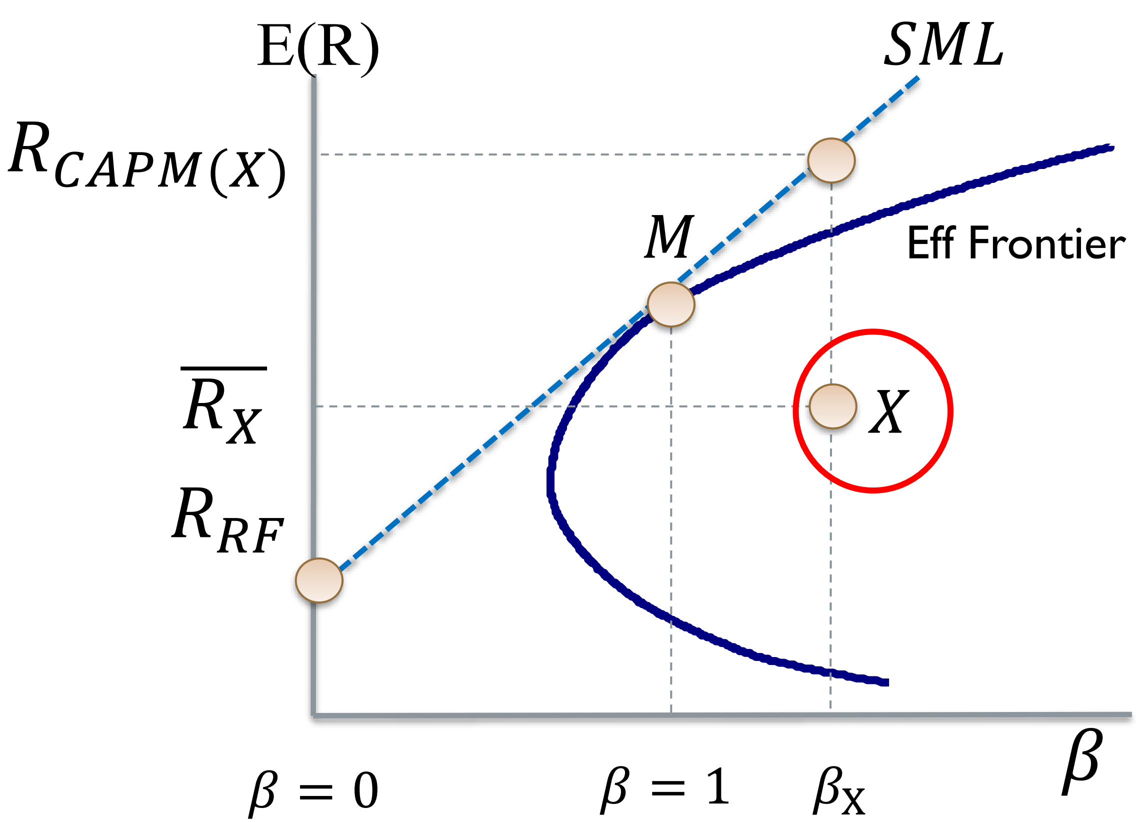
Price of mkt risk

Premium paid for taking on market risk, relative To the market risk

Part left unexplained.

Using CAPM to value X

Using the MPT framework and the SML insights, we can now test whether the price of any asset is fair (i.e. risk adjusted) → if we know its: E(R) and its β (which we find using OLS on past returns)



We can now compare the asset X's true expected return $\overline{R_X}$, (using its current market price) to the price that would ensure a **fair** (risk-adjusted) **return**, which is : $R_{CAPM(X)}$.

We find R_{CAPM} using:

$$\boldsymbol{R_{CAPM(X)} = R_{RF} + \beta_X . (R_M - R_{RF})}$$

And graphically we can see the **current market price** leads to a **lower than fair return** for the risk taken…

As: $\overline{R_X} < R_{CAPMX}$

HENCE k is **overvalued,** and we should see a drop in price {leading to a rise in $\overline{R_X}$) to reflect the relative risk.

CAPM

- <u>Now it is important to keep track of what we imply by the term ***return***.</u>
  - In this context we imply the discounted future income from holding an asset for a certain period.
  - Capital gain returns [i.t.o. relative asset price increases : $\frac{\Delta p_t}{p_t}$]
  - Expected future Dividend payments [$\overline{Div}$]
- This implies that a reduction in the price paid now for the asset implies the asset becoming cheaper relative to future returns - which should raise the expected return of holding it now… as in the future, the same profits are expected than before (this is not changed by the asset price!), which should translate into a higher price for the asset when the expected profit materializes… (but the higher variance of this E(profit) reduces the price.)

Yield Equation

- The fundamental yield equation on the previous slide implies that for two assets with the same level of risk (and implied future capital gains / dividend payments), the asset with a **lower** current price would of course yield a **higher** return.
- This makes the CAPM a profoundly useful analytical tool for assessing how the market prices risk, and can thus be used for calculating a fair (risk-adjusted) asset price.
- Also, if we assume markets are efficient, then the theory can guide individual investment decisions – as it suggests whether an asset is overvalued ($\therefore R_{CAPM(k)} < \overline{R_k}$ → in which case you could **short sell** it) or undervalued (in which case you should **buy** and hold it)…

The CAPM as basis for pricing assets

- The Capital Asset Pricing Model (CAPM) is based on the idea that not all risks affect asset prices. I.e. not all risk are priced in:
  - In fact, **diversifiable** (idiosyncratic or asset-specific) **risk** can be controlled for by combining one asset with another (imperfectly correlated asset) in a portfolio – and thus should **not be compensated for** in the market
    - **Thus asset specific risk is not compensated for in CAPM – with the idea that such risks can be diversified away in the market…**
- This primitive, yet profoundly practical, means of modelling asset prices – gives insight into what kind of risk is incorporated into an asset's price, how to evaluate a portfolio's riskiness relative to its return and also how to conduct portfolio management.

Practical use of CAPM

- Managing desired portfolio risk / return decisions:
  - As a simple example, suppose an investor wishes to earn as much as possible, but cannot afford to lose 10% portfolio value. Assume the market could, in theory, again have a 40% drop in asset value (as was lost roughly on aggregate in 2008). Suppose the current RF is 5%. What portfolio should the investor hold?
    - Using CAPM : $R_P = -10\% = 5\% + \beta.(-40\% - 5\%)$
    - Thus $\beta = 0.33$.
    - The investor should therefore keep only 33% in **M**, while the rest of the portfolio should be kept in **RF**.

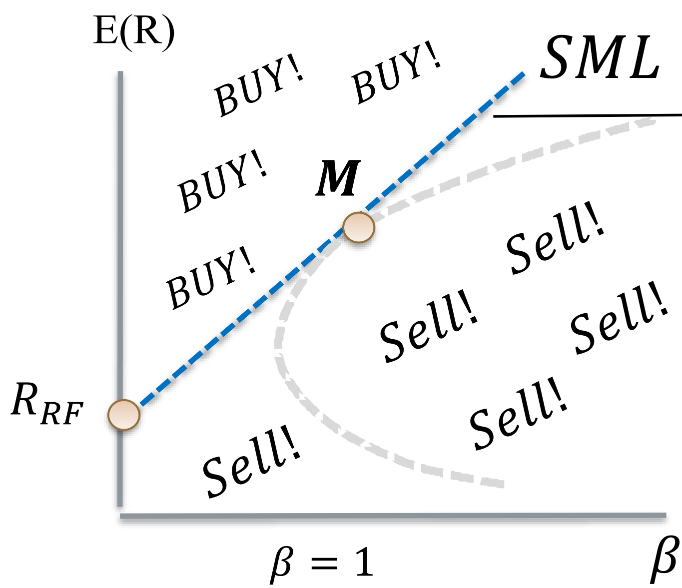
Practical use of CAPM

- <u>Measuring a portfolio's relative performance:</u>
  - Whether a portfolio outperforms the market is **NOT** a sufficient indication of the portfolio's performance!
  - <u>Most uninformed investors can easily get confused by this!</u>
  - If $Return\ A > Return\ B$, it does not imply B is the best portfolio… We need to adjust its returns for **risk!**
  - Thus the risk taken to achieve such a return should be considered, as outperforming the market implies (in theory) that higher beta assets were held, which might lead to significantly lower returns next time!

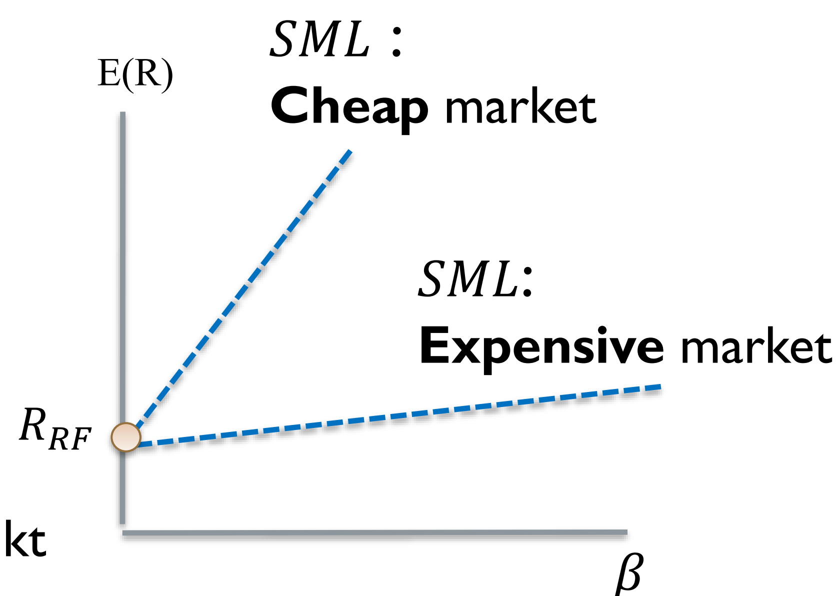
Practical use of CAPM

- <u>Stock selection</u> (as mentioned before):

The fair (risk-adjusted) prices can be calculated in order to assess whether an asset is over- / undervalued given its β



In equilibrium : all assets should lie on the **SML**



- <u>Timing of the market:</u>

Steep SML: Very risk-averse mkt

And thus high returns for holding risk = cheap asset mkt


I'VE GOT A STOCK HERE THAT COULD REALLY EXCEL
REALLY EXCEL?
EXCEL?
SELL?
SELL?
SELL
SELL!
SELL!
SELL!
SELL!
SELL!
THIS IS MADNESS! I CAN'T TAKE ANYMORE
GOOD BYE!
GOOD BYE?
BYE?
Buy?
Buy!
Buy!
BUY!!
Buy!
BUY!
I'VE GOT A STOCK HERE...

Practical use of CAPM

- **Ease of computation:**
  - The parameters of the CAPM can relatively easily be calculated using historical data and OLS techniques – and therefore its applicability lies in its simplicity.
  - More technical and intricate models have subsequently been developed, but the attraction of CAPM remains its simplicity.
- **Classification of funds and assets in terms of the relative riskiness**
  - CAPM provides a theoretic motivation and simple means of comparing different assets and / or portfolios in terms of their expected future risk.
  - A **high-Beta** stock can be considered one that tends to gain more than average in bull markets, and lose more than average in bear markets (and vice versa). This is a useful classification and is engraved in finance lingo!

High Correlations locally:
Average Weekly Correlations (6 months): Top 40 Stocks

Problems with the CAPM approach

- **First Problem**: <u>definition of the parameters M & RF</u>.
  - It is vital to correctly identify an appropriate market index, as its return is used as a benchmark to measure the relative market risk premium. If the broad index return is not a good indication – **betas** will be misleading
  - Also by definition no asset is **truly risk-free.** In particular, holding a government bond has some inherent risk attached to it…
    - **Sovereign Default** (today this happens by currency devaluation or haircuts)
    - **Risk of inflation**
    - **Interest rate risk** (i.e. a change in the rate of interest could lower the value of the bond – returned to later)
    - **Exchange rate risk** (having a Rand –denominated bond could be risky if you are an American investor)

Problems with the CAPM approach

- Asset returns are **not normal** in practice and do not behave exactly (or even remotely) as (especially recent) history suggests:
  - As was seen several times in stock market history, asset returns could vary wildly at times – at levels completely unrelated to the past.
  - The past is not necessarily a good indication of a stock's future returns… This is especially true for new firms' stocks, or firms that experience unpredicted shocks.
  - Of course, seen as CAPM's results are purely extrapolated from past returns – it may be misleading!

Problems with the CAPM approach

- Beta may not be stable.
  - Another major problem may be that Betas are not stable.
  - Asset's returns may co-vary completely differently to that of the market's returns in a recession than it would in an expansion…
  - Thus it may experience modest gains in a bull market, but more intense downturns in a bear market – where in such a scenario β is a dangerous indicator!!
  - Studies have also shown that β values strongly depend on the time horizon of an asset's returns (daily, weekly, monthly, etc).
  - Betas can also change / adapt over time as a firm matures / ages / reaches new agreements.

Problems with the CAPM approach

- Weak definition of **risk.**
  - Recall that we defined risk as being uncertainty about returns – i.e. the probability that the actual returns may differ from the expected returns.
  - We then use the historic **variance** of the asset's returns as a proxy…
  - The problem with this approach to defining risk is that the **variance** is a **squared term** and treats **positive and negative variation as equal** in contributing to an asset's revealed risk.
  - But we know that an asset's risk to an investor is clearly its ability to vary in ***negative territory*** (no investor will complain if it varies positively from what was expected!)

Problems with the CAPM approach

- A good risk measure should rather provide us with an indication of the asset's predicted potential for producing returns **below** some benchmark.
- It should also account for the potential **severity** of such below expected return–periods.
- **<u>Alternative risk measures:</u>**
  - **<u>Semi-variance</u>** : this implies only accounting for the variance of **below** benchmark returns.
  - **<u>Relative lower partial moments</u>**: this studies again only below benchmark returns as contributing to the risk measure, but with the added twist of taking into account the **size** of the shortfall.
    - Again, however, this fails to take into account that investors **do not** experience **linearity** in their disutility of losses…

CAPM today

- Despite the limitations of the CAPM theory and other risk-assessment techniques that are derived from MPT – it is still widely used as **a** (again, not **the**) very useful quantitative tool to assess the relative performance of stocks and portfolios.
- In fact: Beta, Alpha and other Greek names have become terms commonly used by investors and investment institutions in describing, comparing and analysing stock and portfolio returns.
- As mentioned earlier, its simplicity and non-technical nature allows it to be used widely as a first look at the data.
- Certain institutions in SA (such as, e.g., Morningstar) provide frequently updated reports on company and sector **betas**, **alphas** and other standard measures of relative valuation.

Valuations in isolation is dangerous

- In practice, fund managers of course do not only consider price valuations when deciding whether or not to buy a stock.
- For the same reason a home-buyer would not only consider the price of a home before buying.
  - Other factors such as measures of the company's balance-sheet quality, profitability, return generating potential, recent price trends, macroeconomic exposures, etc. all factor into whether a stock should be bought or sold.
  - CAPM is a **single factor approach**, where movement relative to the market is the only consideration. While limited, this provides us with a good starting point to valuing stocks objectively relative to other stocks

Valuations in isolation is dangerous

- We will later in the course return to the topic of moving from a single factor model (only looking at returns relative to the market) to a multi-factor systematic portfolio.

- This aims to chip away at the part that is left unexplained by the CAPM (i.e. the alpha) – in order to systematize that which can be explained by external factors.

THE END

- Next session we will be looking at how we could use the CAPM theory in practice.
  - We will use the guidelines provided by Cadiz investment group and Morningstar on how to implement their analyses using these theoretic models.
- We will also look at how the CAPM has fared in empirical evaluations of its usefulness, and also mention some extensions to the traditional CAPM